MADE EASY & NEXT IAS GROUP

PRESENT

MENNIT

NEET | IIT-JEE | FOUNDATION

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JEE (MAIN) 2022

Test Date: 25th July 2022 (Second Shift)

PAPER-1

Questions with Solutions

Time : 3 Hours **Maximum Marks: 300**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

IMPORTANT INSTRUCTIONS:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts**. **Part-A** is *Physics*, **Part-B** is *Chemistry* and **Part-C** is *Mathematics*. Each part has only two sections: **Section-A** and **Section-B**.
4. **Section – A :** Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** contains 20 multiple choice questions which have only one correct answer. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (1 – 10)** contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which ONLY ONE option is correct.

Q.1 In AM modulation, a signal is modulated on a carrier wave such that maximum and minimum amplitudes are found to be 6V and 2V respectively. The modulation index is :

(a) 100% (b) 80%

(c) 60% (d) 50%

1. (d)

Modulation index = $\frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{6-2}{6+2} = 0.5 \approx 50\%$

Q.2 The electric current in a circular coil of 2 turns produces a magnetic induction B_1 at its centre. The coil is unwound and is rewound into circular coil of 5 turns and at its centre. The coil is unwound and is rewound into a circular coil of 5 turns and the same current produces a magnetic induction B_2 at its centre. The ratio of $\frac{B_2}{B_1}$ is

(a) $\frac{5}{2}$ (b) $\frac{25}{4}$

(c) $\frac{5}{4}$ (d) $\frac{25}{2}$

2. (b)

$B_1 = \frac{N_1\mu_0 I}{2R}$ given $N_1 = 2$

When new loop is made the length of wire remains same, so

$$N_2 \times 2\pi r = N_1 \times 2\pi R \Rightarrow r = \frac{N_1 R}{N_2}$$

$$\Rightarrow \quad B_2 = \frac{N_2\mu_0 I}{2r} = \frac{N_2^2\mu_0 I}{2N_1 R} = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{N_1\mu_0 I}{2R}\right) = \left(\frac{N_2}{N_1}\right)^2 B_1$$

$$\Rightarrow \quad \frac{B_2}{B_1} = \left(\frac{N_2}{N_1}\right)^2 = \frac{25}{4}$$

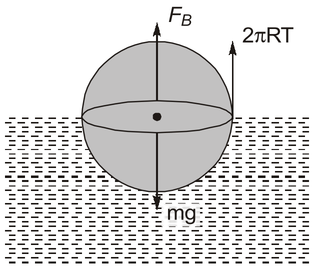
Q.3 A drop of liquid of density ρ is floating half immersed in a liquid of density σ and surface tension 7.5×10^{-4} Ncm^{-1}. The radius of drop in cm will be : ($g = 10$ ms^{-2})

(a) $\frac{15}{\sqrt{(2\rho - \sigma)}}$ (b) $\frac{15}{\sqrt{(\rho - \sigma)}}$

(c) $\frac{3}{2\sqrt{(\rho - \sigma)}}$ (d) $\frac{3}{20\sqrt{(2\rho - \sigma)}}$

3. (a)

Since liquid drop is in equilibrium , so



$$mg = F_B + 2\pi RT$$

$$\Rightarrow \quad \frac{4}{3}\pi R^3 \rho g = \frac{2}{3}\pi R^3 \sigma g + 2\pi RT$$

$$\Rightarrow \quad \frac{1}{3}R^2(2\rho - \sigma)g = T$$

$$\Rightarrow \quad R^2 = \frac{3T}{(2\rho - \sigma)g} = \frac{3 \times 7.5 \times 10^{-2}}{(2\rho - \sigma) \times 10}$$

$$\Rightarrow \quad R = \frac{3T}{(2\rho - \sigma)g} = \frac{15}{\sqrt{(2\rho - \sigma)}} \times 10^{-2} m = \frac{15}{\sqrt{(2\rho - \sigma)}} cm$$

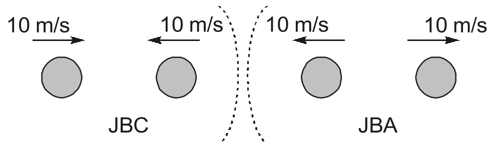
Q.4 Two billiard balls of mass 0.05 kg each moving in opposite directions with 10 ms^{-1} collide and rebound with the same speed. If the time duration of contact is $t = 0.005$ s. then what is the force exerted on the ball due to each other?

(a) 100 N (b) 200 N
(c) 300 N (d) 400 N

4. (b)

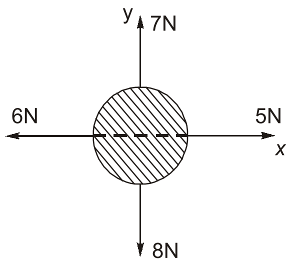
Δp = Change in momentum of each ball

$\Delta p = 2mv = 2 \times 0.05 \times 10 = 1$ Ns



$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t} = \frac{2 \times 0.05 \times 10}{0.005} = 200\ N$$

Q.5 For a free body diagram shown in the figure, the four forces are applied in the 'x' and 'y' directions. What additional force must be applied and at what angle with positive x-axis so that the net acceleration of body is zero?



(a) $\sqrt{2}N, 45°$ (b) $\sqrt{2}N, 135°$
(c) $\frac{2}{\sqrt{3}}N, 30°$ (d) $2N, 45°$

5. (a)

Let additional force is $\vec{F}$, so

$$\vec{F} + 5(+\hat{i}) + 6(-\hat{i}) + 7(+\hat{j}) + 8(-\hat{j}) = \vec{0}$$

$$\Rightarrow \quad \vec{F} - (\hat{i} + \hat{j}) = \vec{0} \Rightarrow \vec{F} = \hat{i} + \hat{j} \Rightarrow F = \sqrt{2}N, 45°$$

Q.6 Capacitance of an isolated conducting sphere of radius R_1 becomes n times when it is enclosed by a concentric conducting sphere of radius R_2 connected to earth. The ratio of their radii $\left(\frac{R_2}{R_1}\right)$ is :

(a) $\frac{n}{n-1}$ (b) $\frac{2n}{2n+1}$
(c) $\frac{n+1}{n}$ (d) $\frac{2n+1}{n}$

6. (a)

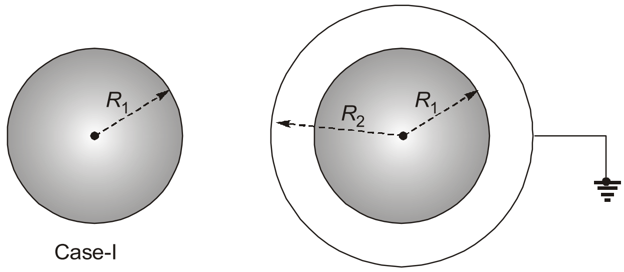
$$C_1 = 4\pi R_1 \text{ and } C_2 = \frac{4\pi R_2 R_1}{R_2 - R_1} = \frac{R_2 C_1}{R_2 - R_1}$$

$$\Rightarrow \quad \frac{C_2}{C_1} = \frac{4\pi R_2 R_1}{R_2 - R_1} = \frac{R_2}{R_2 - R_1} = n$$

$$\Rightarrow \quad nR_2 - nR_1 = R_2$$

$$\Rightarrow \quad (n-1)R_2 = nR_1$$

$$\Rightarrow \quad \frac{R_2}{R_1} = \frac{n}{n-1}$$



Case-I   Case-II

Q.7 The ratio of wavelengths of proton and deuteron accelerated by potential V_p and V_d is $1 : \sqrt{2}$. Then, the ratio of V_p to V_d will be :

(a) 1 : 1
(b) $\sqrt{2}$: 1
(c) 2 : 1
(d) 4 : 1

7. (d)

According to de-Broglie's hypothesis, we can write

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\,meV}} \qquad \Rightarrow \frac{\lambda_P}{\lambda_D} = \frac{\sqrt{2m_D eV_D}}{\sqrt{2m_P eV_P}} = \frac{1}{\sqrt{2}} \qquad \Rightarrow \frac{2m_P V_D}{m_P V_P} = \frac{1}{2} \Rightarrow \frac{V_P}{V_D} = 4:1$$

Q.8 For an object placed at a distance 2.4 m from a lens, a sharp focused image is observed on a screen placed at a distance 12 cm from the lens. A glass plate of refractive index 1.5 and thickness 1 cm is introduced between lens and screen such that the glass plate plane faces parallel to the screen. By what distance should the object be shifted so that a sharp focused image is observed again on the screen?

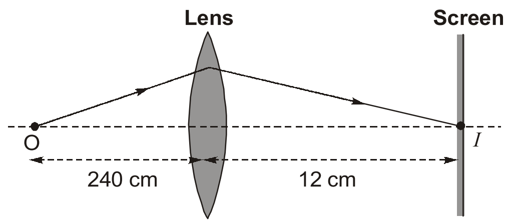
(a) 0.8 m
(b) 3.2 m
(c) 1.2 m
(d) 5.6 m

8. (b)

When glass slab is not put in between lens and screen

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{12} - \frac{1}{-240} = \frac{1}{f} \qquad ...(i)$$



When glass slab is put in between lens and screen

$$\text{Shift} = t\left(1 - \frac{1}{\mu}\right) = 1\left(1 - \frac{1}{1.5}\right) = \frac{1}{3}\text{cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

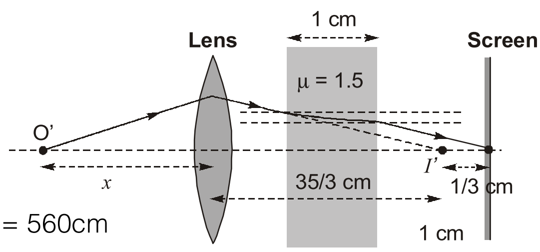
$$\Rightarrow \quad \frac{3}{35} - \frac{1}{-x} = \frac{1}{f} \qquad ...(ii)$$

With the help of equations (*i*) and (*ii*), we can write

$$\frac{3}{35} - \frac{1}{-x} = \frac{1}{12} - \frac{1}{-240}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{12} + \frac{1}{240} - \frac{3}{35} = \frac{735 - 720}{240 \times 35} \Rightarrow x = \frac{240 \times 35}{15} = 560\text{cm}$$

$$\Rightarrow \quad |\Delta u| = 560 - 240 = 320 \text{ cm} = 3.2 \text{ m}$$



Q.9 Light wave traveling in air along x-direction is given by

$E_y = 540 \sin \pi \times 10^4 (x - ct)\ Vm^{-1}$. Then, the peak value of magnetic field of wave will be

(Given $c = 3 \times 10^8\ ms^{-1}$)

(a) $18 \times 10^{-7}\ T$ (b) $54 \times 10^{-7}\ T$

(c) $54 \times 10^{-8}\ T$ (d) $18 \times 10^{-8}\ T$

9. **(a)**

$$C = \frac{E_0}{B_0} \Rightarrow B_0 = \frac{E_0}{c} = \frac{540}{3 \times 10^8} = 18 \times 10^{-8}\ T$$

Q.10 When you walk through a metal detector carrying a metal object in your pocket, it raises an alarm. This phenomenon works on :

(a) Electromagnetic induction (b) Resonance in ac circuits

(c) Mutual induction in ac circuits (d) Interference of electromagnetic waves

10. **(b)**

This phenomenon works on resonance in ac circuit.

Q.11 An electron with energy 0.1 keV moves at right angle to the earth's magnetic field of $1 \times 10^{-4} Wbm^{-2}$. The frequency of revolution of the electron will be

(Take mass of electron = 9.0×10^{-31}kg)

(a) 1.6×10^5 Hz (b) 5.6×10^5 Hz

(c) 2.8×10^6 Hz (d) 1.8×10^6 Hz

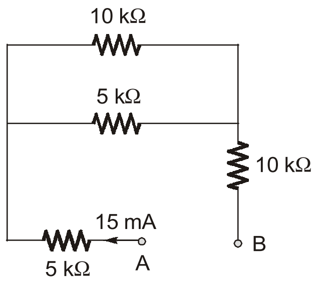
11. **(c)**

$$f = \frac{qB}{2\pi m} = \frac{1.6 \times 10^{-19} \times 1.0 \times 10^{-4}}{2 \times 3.14 \times 9 \times 10^{-31}}$$

$$= 0.028 \times 10^8 \text{ Hz} = 2.8 \times 10^6 \text{ Hz}$$

Q.12 A current of 15 mA flows in the circuit as shown in figure.

The value of potential difference between the pointes A and B will be :



(a) 50 V (b) 75 V

(c) 150 V (d) 275 V

12. **(d)**

$$R_{AB} = 5 + (10/5) + 10 = 15 + \frac{10 \times 5}{10 + 5} = 15 + \frac{10}{3} = \frac{55}{3}$$

$$V_{AB} = R_{AB}\ I = \left(\frac{55}{3} \times 10^3\right) \times (15 \times 10^{-3}) = 275 \text{ V}$$

Q.13 The length of a seconds pendulum at a height $h = 2R$ from earth surface will be :

(Given R = Radius of earth and acceleration due to gravity at the surface of earth, $g = \pi^2\ ms^{-2}$)

(a) $\frac{2}{9}$ m (b) $\frac{4}{9}$ m

(c) $\frac{8}{9}$ m (d) $\frac{1}{9}$ m

13. (d)

As we know that $g_P = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{R+2R}\right)^2 = \frac{g}{9}$, so

$$T = 2\pi\sqrt{\frac{l}{g_p}} \Rightarrow l = \frac{g_pT^2}{4\pi^2} = \frac{\frac{g}{9}\times(2)^2}{4\pi^2} = \frac{1}{9}\text{m}$$

Q.14 Sound travels in a mixture of two moles of helium and n moles of hydrogen. If rms speed of gas molecules in the mixture is $\sqrt{2}$ times the speed of sound, then the value of n will be :

(a) 1 (b) 2

(c) 3 (d) 4

14. (b)

as we know that $v = \sqrt{\frac{\gamma RT}{M}} \Rightarrow$ Speed of sound in a gas, and

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow \text{Rms speed of gas molecules, so}$$

$$v_{rms} = \sqrt{2}v \Rightarrow \sqrt{\frac{3\,RT}{M}} = \sqrt{2}\sqrt{\frac{\gamma\,RT}{M}} \Rightarrow \gamma = \frac{3}{2}$$

According formula of specific heat ratio of mixture, we can write

$$\frac{n_1+n_2}{\gamma} = \frac{n_1}{\gamma_1}+\frac{n_2}{\gamma_2} \Rightarrow \frac{2+n}{\frac{3}{2}} = \frac{2}{\frac{5}{3}}+\frac{n}{\frac{7}{5}} \Rightarrow \frac{4+2n}{3} = \frac{6}{5}+\frac{5n}{7} \Rightarrow 140+70n = 126+75n$$

$\Rightarrow$ $5n = 14 \Rightarrow n = 2.8$

Q.15 Let η_1 is the efficiency of an engine at $T_1 = 447°C$ and $T_2 = 147°C$ while η_2 is the efficiency at $T_1 = 947°C$ and $T_2 = 47°C$. The ratio $\frac{\eta_1}{\eta_2}$ will be :

(a) 0.41 (b) 0.56

(c) 0.73 (d) 0.70

15. (b)

as we know that $\eta = 1-\frac{T_2}{T_1}$, so

$$\frac{\eta_1}{\eta_2} = \frac{1-\frac{T_{12}}{T_{11}}}{1-\frac{T_{22}}{T_{21}}} = \frac{1-\frac{T_{12}}{T_{11}}}{1-\frac{T_{22}}{T_{21}}} = \frac{1-\frac{420}{720}}{1-\frac{320}{1220}} = \frac{1-0.58}{1-0.26}$$

$\Rightarrow$ $\frac{\eta_1}{\eta_2} = \frac{0.42}{0.74} \approx 0.56$

Q.16 An object is taken to a height above the surface of earth at a distance $\frac{5}{4}R$ from the centre of the earth. Where radius of earth, $R = 6400$ km. The percentage decrease in the weight of the object will be :

(a) 36% (b) 50%

(c) 64% (d) 25%

16. (a)

as we know that $$gP = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{\frac{5}{4}R}\right)^2 = \frac{16g}{25}$$

$\Rightarrow$ $$\Delta g = g - g_P = \frac{9g}{25}$$

The percentage decrease in the weight of the object = $\frac{\Delta g}{g} \times 100 = 36\%$

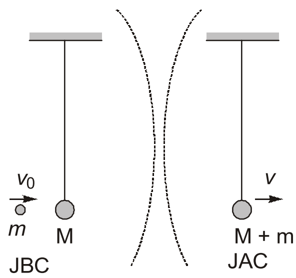
Q.17 A bag of sand of mass 9.8 kg is suspended by a rope. A bullet of 200 *g* travelling with speed 10 ms^{-1} gets embedded in it, then loss of kinetic energy will be :

(a) 4.9 J (b) 9.8 J
(c) 14.7 J (d) 19.6 J

17. (b)

$$v = \frac{mv_0}{M+m} \Rightarrow K_f = \frac{1}{2}(M+m)v^2 = \frac{mv_0^2}{2(M+m)}$$

$$K_t = \frac{mv_0^2}{2}$$



Loss in kinetic energy = $\Delta K = K_i - K_f$

$\Rightarrow$ $$\Delta K = \frac{1}{2}mv_0^2 - \frac{m^2v_0^2}{2(M+m)} = \frac{1}{2}mv_0^2\left(1 - \frac{m}{M+m}\right)$$

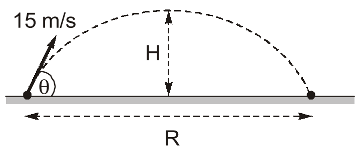
$\Rightarrow$ $$\Delta K = \frac{1}{2}mv_0^2\left(\frac{M}{M+m}\right) = \frac{1}{2}\times 0.2 \times 100\left[\frac{9.8}{10}\right] = 9.8\text{ J}$$

Q.18 A ball is projected from the ground with a speed 15 ms^{-1} at an angle θ with horizontal so that its range and maximum height are equal. Then 'tan θ' will be equal to :

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 2 (d) 4

18. (d)

$$H = \frac{u^2\sin^2\theta}{2\text{ g}} \text{ and } R = \frac{u^2\sin 2\theta}{\text{g}}$$



According to quesiton

$$R = H \Rightarrow \frac{u^2\sin^2\theta}{2\text{g}} = \frac{2u^2\sin\theta\cos\theta}{\text{g}} \Rightarrow \tan\theta = 4$$

Q.19 The maximum error in the measurement of resistance, current and time for which current flows in an electrical circuit are 1%, 2% and 3% respectively. The maximum percentage error in the detection of the dissipated heat will be :

(a) 2 (b) 4
(c) 6 (d) 8

19. (d)

$$H = I^2Rt$$

% error in $H = \frac{\Delta H}{H}\times 100 = 2\left(\frac{\Delta I}{I}\right)\times 100 + \left(\frac{\Delta R}{R}\right)\times 100 + \left(\frac{\Delta t}{t}\right)\times 100 = 2\times 2 + 1 + 3 = 8\%$

Q.20 Hydrogen atom from exited state comes to the ground state by emitting a photon of wavelength λ. The value of principal quantum number 'n' of the excited state will be :

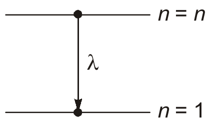
(R : Rydberg constant)

(a) $\sqrt{\dfrac{\lambda R}{\lambda - 1}}$ (b) $\sqrt{\dfrac{\lambda R}{\lambda R - 1}}$

(c) $\sqrt{\dfrac{\lambda}{\lambda R - 1}}$ (d) $\sqrt{\dfrac{\lambda R^2}{\lambda R - 1}}$

20. **(b)**

According to definition of wave number, we can write



$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow 1 - \frac{1}{n^2} = \frac{1}{\lambda R}$$

$$\Rightarrow \quad \frac{1}{n^2} = 1 - \frac{1}{\lambda R} = \frac{\lambda R - 1}{\lambda R} \Rightarrow n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

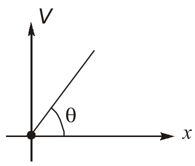
SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q.1 A particle is moving in a straight line such that its velocity is increasing at 5 ms^{-1} per meter. The acceleration of the particle is ________ ms^{-2} at a point where its velocity is 20 ms^{-1}.

1. **(100)**



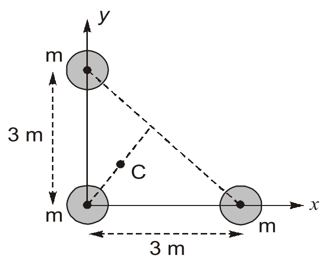
$$\tan\theta = \frac{dv}{dx} = 5(\text{m/s})/\text{m}$$

$$\Rightarrow \quad dv = 5dx \Rightarrow \frac{dv}{dt} = 5\frac{dx}{dt}$$

$$\Rightarrow \quad a = 5v = 5 \times 20 = 100 \text{ m/s}^2$$

Q.2 Three identical spheres each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 3 m each. Taking point of intersection of mutually perpendicular sides as origin, the magnitude of position vector of centre of mass of the system will be $\sqrt{x}$ m. The value of x is ________.

2. **(2)**



$$y_c = y_c = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{m \times 0 + m \times 0 + m \times 3}{3m} = 1 \text{ m}$$

$$OC = X\sqrt{x_0^2 + Y_c^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2}m$$

Q.3 A block of ice of mass 120g at temperature 0°C is put in 300g of water at 25°C . The xg of ice melts as the temperature of the water reaches 0°C. The value of x is _______.

[Use specific heat capacity of water = 4200 $Jkg^{-1}K^{-1}$, Latent heat of ice = 3.5×10^5 $Jk g^{-1}$]

3. (90)

Using concept of calorimeter, we can write Gain in heat = Loss in heat

$$x \times L = mc\Delta T \Rightarrow x \times 3.5 \times 10^5 = \frac{300}{1000} \times 25 \times 4200$$

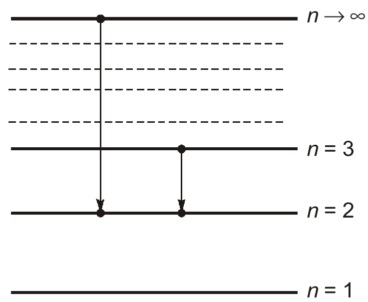
$$\Rightarrow \quad x = \frac{300 \times 25 \times 4200}{1000 \times 3.5 \times 10^5} = \frac{3 \times 25 \times 42}{35 \times 1000} = \frac{90}{1000} \text{ kg} = 90 \text{ gm}$$

Q.4 $\frac{x}{x+4}$ is the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its

(i) third permitted energy level to the second level and

(ii) the highest permitted energy level to the second permitted level.

The value of x will be ______.



4. (5)

According to definition of atomic energy, we can write

$$E_1 = E_0\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5E_0}{36}, \text{ and } E_2 = E_0\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$\frac{E_1}{E_2} = \frac{5}{9} \Rightarrow \frac{x}{x+4} = \frac{5}{5+4} \Rightarrow x = 5$$

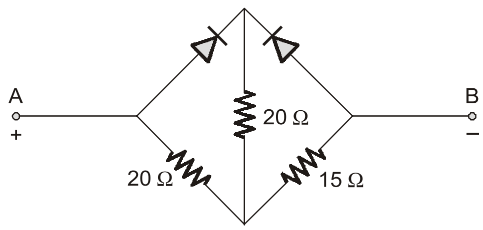
Q.5 In a potentiometer arrangement, a cell of emf 1.20 V gives a balance point at 36cm length of wire. This cell is now replaced by another cell of emf 1.80 V. The difference in balancing length of potentiometer wire in above conditions will be _______cm.

5. (18)

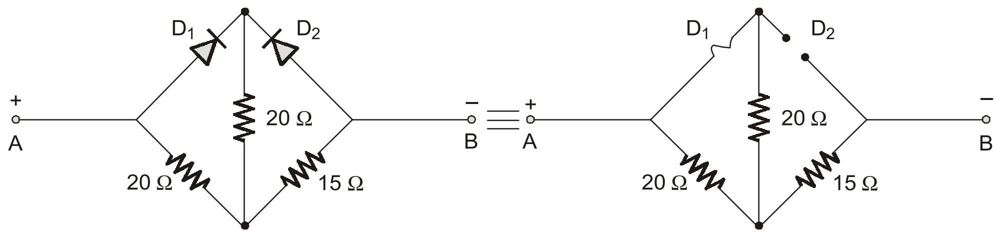
$$V = Kl \Rightarrow \frac{V_2}{V_1} = \frac{l_2}{l_1} \Rightarrow \frac{l_2}{36} = \frac{1.80}{1.20} = \frac{3}{2} \Rightarrow l_2 = 54\text{cm}$$

$$\Delta l = l_2 - l_1 = 54 - 36 = 18 \text{ cm}$$

Q.6 Two ideal diodes are connected in the network as shown is figure. The equivalent resistance between A and B is _______ Ω.



6. (25)



Here diode-D_1 is forced biased and diode-D_2 is reversed biased, so

$$R_{AB} = (20 / 20) + 15 = 25\Omega$$

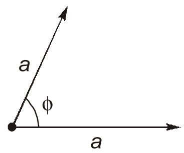
Q.7 Two waves executing simple harmonic motions travelling in the same direction with same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is _______ (degree).

7. (60)

$$A^2 = a_1^2 + a_2^2 + 2\,a_1\,a_2\cos\phi = 2a^2 + 2a^2\cos\phi = 4a^2\cos^2\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad A = 2a\cos\left(\frac{\phi}{2}\right) = \sqrt{3}a$$

$$\Rightarrow \quad \cos\left(\frac{\phi}{2}\right) = \frac{\sqrt{3}}{2} \Rightarrow \frac{\phi}{2} = 30^\circ \Rightarrow 60^\circ$$



Q.8 Two parallel plate capacitors of capacity C and $3C$ are connected in parallel combination and charged to a potential difference 18 V. The battery is then disconnected and the space between the plates of the capacitor of capacity C is completely filled with a material of dielectric constant 9. The final potential difference across the combination of capacitors will be ________ V.

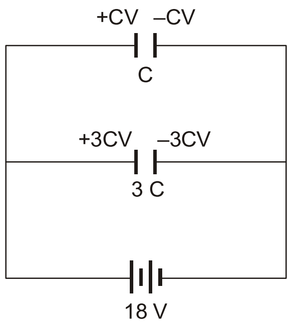
8. (6)

$$\frac{CV+q}{9C} + \frac{-3CV+q}{3C} = 0$$

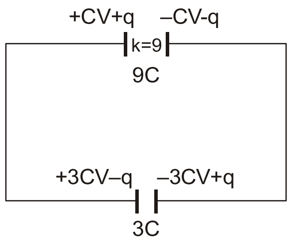
$$\Rightarrow \quad CV + q - 9CV + 9q = 0$$

$$\Rightarrow \quad q = \frac{8\,CV}{4} = 2\,CV$$

$$V_{AB} = \frac{CV+q}{9C} = \frac{3CV}{9C} = \frac{V}{3} = \frac{18}{3} = 6 \text{ volt}$$



Initial Condition



Final Condition

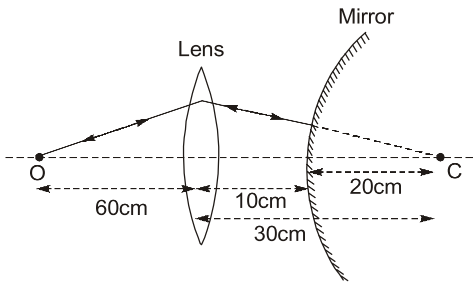
Q.9 A convex lens of focal length 20cm is placed in front of a convex mirror with principal axis coinciding each other. The distance between the lens and mirror is 10 cm. A point object is placed on principal axis at a distance of 60cm from the convex lens. The image formed by combination coincides the object itself. The focal length of the convex mirror is _______ cm.

9. (10)

For refraction of light at lens $\frac{1}{v} - \frac{1}{-60} = \frac{1}{20}$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{20} - \frac{1}{60} \Rightarrow \frac{1}{v} = \frac{3-1}{60} = \frac{1}{30}$$

$$\Rightarrow \quad v = 30 \text{ cm}$$



If image has to be formed at object itself then light ray should retrace its path. Hence after refraction at lens, it must strikes normally to the mirror

$$R_M = 20 \text{ cm} \Rightarrow \text{Radius of curvature of mirror}$$

$$F_m = 10 \text{ cm} \Rightarrow \text{focal of mirror}$$

Q.10 Magnetic flux (in weber) in a closed circuit of resistance 20Ω varies with time $t\,(s)$ as $\phi = 8t^2 - 9t + 5$. The magnitude of the induced current at $t = 0.25$ s will be ______ mA.

10. (250)

$$e_{in} = \left|\frac{d\phi}{dt}\right| = |16t - 9|\; e_{in} = \left|16 \times \frac{1}{4} - 9\right| = 5 \text{ volt}$$

$$I = \frac{e_{in}}{R} = \frac{5}{20}A = \frac{5}{20} \times 1000 \text{ mA} = 250 \text{ mA}$$

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q.1 Match **List I** with **List II**.

List I (molecule)	List II (hybridization; shape)
A. XeO_3	(I) sp^3d; linear
B. XeF_2	(II) sp^3; pyramidal
C. $XeOF_4$	(III) sp^3d^3; distorted octahedral
D. XeF_6	(IV) sp^3d^2; square pyramidal

Choose the correct answer from the option given below:

(a) A-(II), B-(I), C-(IV), D-(III)
(b) A-(II), B-(IV), C-(III), D-(I)
(c) A-(IV), B-(II), C-(III), D-(I)
(d) A-(IV), B-(II), C-(I), D-(III)

1. (a)

$XeO_3 \Rightarrow sp^3$, Pyramidal
$XeF_2 \Rightarrow sp^3d^2$, Square pyramidal
$XeF_6 \Rightarrow sp^3d^3$, Distorted octahedral
$XeOF \Rightarrow sp^3d^2$, Square pyramidal

Q.2 Two solutions *A* and *B* are prepared by dissolving 1 g of non- volatile solutes *X* and *Y*, respectively in 1 kg of water. The ratio of depression in freezing points for *A* and *B* is found to be 1 : 4. The ratio of molar masses of *X* and *Y* is

(a) 1 : 4
(b) 1 : 0.25
(c) 1 : 0.20
(d) 1 : 5

2. (b)

$$\Delta T_f = i \times k_t \times M$$

$$\frac{(\Delta T_f)_x}{(\Delta T_f)_y} = \frac{iK_fM_x}{iK_fM_Y} = \frac{M_x}{M_Y}$$

Q.3 K_{a_1}, K_{a_2} and K_{a_3} are the respective ionization constants for the following reaction (*a*), (*b*) and (*c*).

1. $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$
2. $HC_2O_4^+ \rightleftharpoons H^- + C_2O_4^{2-}$
3. $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$

The relationship between K_{a_1}, K_{a_2} and K_{a_3} is given as

(a) $K_{a_3} = K_{a_1} + K_{a_2}$
(b) $K_{a_3} = K_{a_1} - K_{a_2}$
(c) $K_{a_3} = K_{a_1} / K_{a_2}$
(d) $K_{a_3} = K_{a_1} / K_{a_2}$

3. (d)

(I) $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$

(II) $HC_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-}$

(I) + (II); $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$

$K_{a_3} = K_{a_1} \times K_{a_2}$

Q.4 The molar conductivity of a conductivity cell filled with 10 moles of 20 mL NaCl solution is Λ_{m1} and that of 20 moles another identical cell heaving 80mL NaCl solution is Λ_{m2}. The conductivities exhibited by these two cells are same.

The relationship between Λ_{m2} and Λ_{m1} is

(a) $\Lambda_{m2} = 2\Lambda_{m1}$

(b) $\Lambda_{m2} = \Lambda_{m1} / 2$

(c) $\Lambda_{m2} = \Lambda_{m1}$

(d) $\Lambda_{m2} = 4\Lambda_{m1}$

4. **(a)**

$$\wedge_M = \frac{K \times 1000}{M} \qquad M_2 = \frac{20}{(80/1000)}$$

$$\frac{\wedge_{M_1}}{\wedge_{M_2}} = \frac{M_2}{M_1} \qquad M_1 = \frac{10}{(20/1000)}$$

$$\frac{\wedge_{M_1}}{\wedge_{M_2}} = \frac{1}{2} \Rightarrow \wedge_{M_2} = 2\wedge_{M_1}$$

Q.5 For micelle formation, which of the following statements are correct?

1. Micelle formation is an exothermic process.
2. Micelle formation is an endothermic process.
3. The entropy change is positive.
4. The entropy change is negative.

Which of the above statements is/are correct?

(a) 1 and 4 only

(b) 1 and 3 only

(c) 2 and 3 only

(d) 2 and 4 only

5. **(b)**

During micelle formation, ΔS is positive.

Micelle formation is endothermic at low temperature and exothermic at high temperature.

At room temperature $\Rightarrow$ Micelle formation is endothermic.

Q.6 The first ionization entahalpies of Be, B, N and O follow the order

(a) O < N < B < Be

(b) Be < B < N < O

(c) B < Be < N < O

(d) B < Be < O < N

6. **(d)**

Ionization enthalpy: B < Be < O < N

IE of N > O [Due to half- filled configuration]

IE of Be > B [Due to penetration effect]

Q.7 Given below are two statements.

Statement I: Pig iron is obtained by heating cast iron with scrap iron.

Statement II: Pig iron has a relatively lower carbon content than that of cast iron. In the light of the above statements, choose the correct answer form the options given below.

(a) Both Statement I and Statement II are correct.

(b) Both Statement I and Statement II are not correct.

(c) Statement I is correct but Statement II is not correct

(d) Statement I is not correct but Statement II is correct

7. (b)
Cast iron is formed by pig iron and scrap iron.
Pig Iron $\rightarrow$ Carbon 4%
Cast Iron $\rightarrow$ Carbon 3%

Q.8 High purity (> 99.95%) dihydrogen is obtained by
(a) Reaction of zinc aqueous alkali.
(b) Electrolysis of acidified water using platinum electrodes.
(c) Electrolysis of warm aqueous barium hydroxide solution between nickel electrodes.
(d) Reaction of zinc with dilute acid.

8. (c)
High purity (> 99.95%) H_2 gas is obtained by electrolysis of warm aqueous $Ba(OH)_2$ solution using Ni electrode.

Q.9 The correct order of density is
(a) Be > Mg > Ca > Sr
(b) Sr > Ca > Mg > Be
(c) Sr > Be > Mg > Ca
(d) Be > Sr > Mg > Ca

9. (c)

Element	Density (gm / cm^3)
Be	1.84
Mg	1.74
Ca	1.55
Sr	2.63

Q.10 The total number of acidic oxides from the following list is
NO, N_2O, B_2O_3, N_2O_5, CO, SO_3, P_4O_{10}
(a) 3
(b) 4
(c) 5
(d) 6

10. (b)
Acidic oxides: B_2O_3, P_4O_{10}, SO_3, N_2O_5
Neutral oxide: N_2O, NO, CO

Q.11 The correct order of energy of absorption for the following metal complexes is
A : $[Ni(en)_3]^{2+}$, B : $[Ni(NH_3)_6]^{2+}$, C : $[Ni(H_2O)_6]^{2+}$
(a) C < B < A
(b) B < C < A
(c) C < A < B
(d) A < C < B

11. (a)
Stronger ligand will produce more Δ_0, So energy absorbed will be of greater extent.

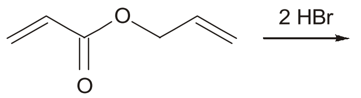
Q.12 Match **List I** with **List II**.

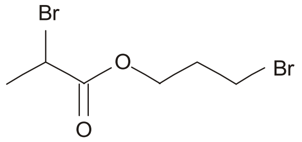
List I (molecule)	**List II (hybridization; shape)**
A. Sulphate	(I) pesticide
B. Fluoride	(II) Bending of bones
C. Nicotine	(III) Laxative effect
D. Sodium arsinite	(IV) Herbicide

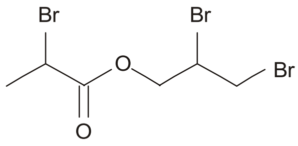
Choose the correct answer from the option given below:
(a) A-(II), B-(III), C-(IV), D-(I)
(b) A-(IV), B-(III), C-(II), D-(I)
(c) A-(III), B-(II), C-(I), D-(IV)
(d) A-(III), B-(II), C-(IV), D-(I)

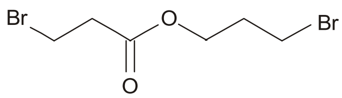
12. **(c)**
Based upon the properties and uses of chemical substances.

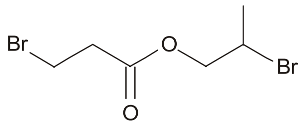
Q.13 Major product of the following reaction is



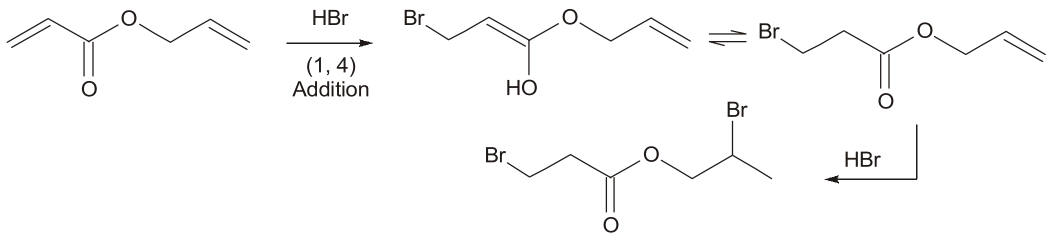
(a)


(b)


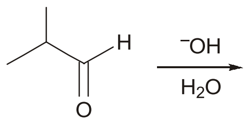
(c)


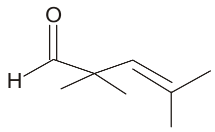
(d)


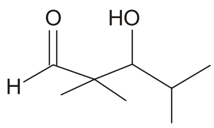
13. **(d)**

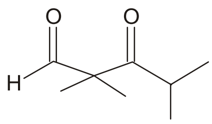


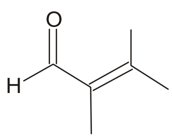
Q.14 What is the major product of the following reaction?



(a)


(b)


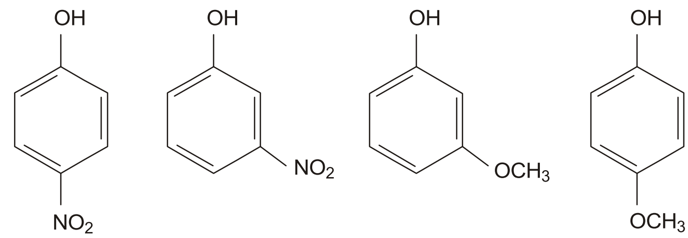
(c)


(d)


14. **(b)**

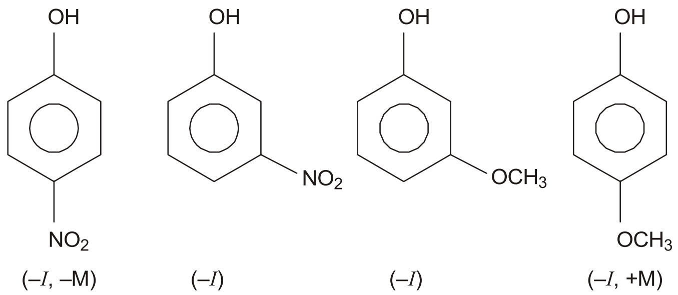
$$2\,CH_3-\underset{}{\overset{H_3C}{\overset{|}{CH}}}-\overset{O}{\overset{\|}{C}}-H \xrightarrow{\bar{O}H} CH_3-\overset{CH_3}{\overset{|}{CH}}-\overset{OH}{\overset{|}{CH}}-\underset{CH_3}{\underset{|}{\overset{CH_3}{\overset{|}{C}}}}-\overset{O}{\overset{\|}{C}}-H$$

Q.15 Arrange the following in decreasing acidic strength.



(a) A > B > C > D (b) B > A > C > D
(c) D > C > A > B (d) D > C > B > A

15. (a)



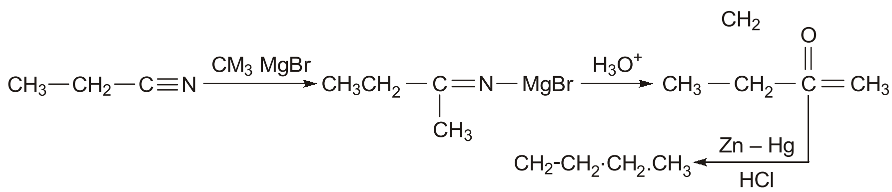
Q.16 $CH_3-CH_2-CN \xrightarrow[\text{Ether}]{CM_3\,MgBr} A \xrightarrow{H_3O^+} B \xrightarrow[\text{HCl}]{\text{Zn – Hg}} C$

The correct structure of *C* is

(a) $CH_3-CH_2-CH_2-CH_3$ (b) $CH_3-CH_2-\overset{\overset{\large O}{\|}}{C}-CH_3$

(c) $CH_3-CH_2-\underset{}{\overset{\overset{\large OH}{|}}{CH}}-CH_3$ (d) $CH_3-CH_2-CH=CH_2$

16. (a)



Q.17 Match **List I** with **List II**.

List I (molecule)	List II (hybridization; shape)
A. Nylon 6.6	(I) Buckets
B. Low density polythene	(II) Non-stick utensils
C. High density polythene	(III) Bristles of brushes
D. Teflon	(IV) Toys

Choose the correct answer from the option given below:

(a) A-(III), B-(I), C-(IV), D-(II) (b) A-(III), B-(IV), C-(I), D-(II)
(c) A-(II), B-(I), C-(IV), D-(III) (d) A-(II), B-(IV), C-(I), D-(III)

17. (b)

Based upon the properties and uses of chemical substances.

Q.18 Glycosidic linkage between *C*1 of α- glucose and *C*2 of β-fructose is found in

(a) Maltose (b) Sucrose
(c) Lactose (d) Amylose

18. (b)

In sucrose, glycosidic linkage is between C_1 of α-glucose and C_2 of β- fructose.

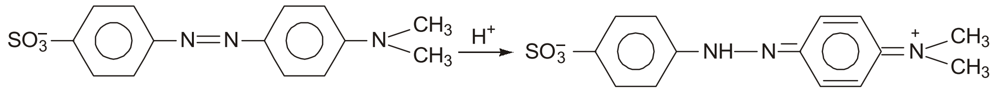
Q.19 Some drugs bind to a site other than the active site of an enzyme. This site is known as

(a) non-active site (b) allosteric site
(c) competitive site (d) therapeutic site

19. (b)
Based upon the properties and uses of chemical substances.

Q.20 In base vs, acid titration, at the end point methyl orange is present as
(a) quinonoid form (b) heterocyclic form
(c) phenolic form (d) benzenoid form

20. (a)



Benzoid form
(Yellow colour)

Quinonoid form
(Pinkish red)

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q.1 56.0 L of nitrogen gas is mixed with excess of hydrogen gas and it is found that 20 L of ammonia gas is produced. The volume of unused nitrogen gas is found to be _____ L.

1. **(46)**

	$N_2(g)$ +	$3H_2(g)$ $\longrightarrow$	$2NH_3(g)$
$t = 0$	$56\ l$	Excess	0
	$-10\ l$		$+20\ l$
Final	$46\ l$		$20\ l$

Vol. of $N_2(g)$ remained unreacted = $46\ l$

Q.2 A sealed flask with a capacity of 2 dm^3 contains 11g of propane gas. The flask is so weak that it will burst if the pressure becomes 2 MPa. The minimum temperature at which the flask will burst is __________°C. [Nearest integer]
(**Given:** R = 8.3 J K^{-1} mol^{-1}. Atomic masses of C and H are 12u and 1u, respectively) (Assume that propane behaves as an ideal gas.)

2. **(1655)**
$V = 2l$, $n = 0.25$ Moles, $P = 2\times10^6 Pa$, $R = 8.314$ J /mol.k
$PV = nRT$
$2 \times 10^6 \times 2 \times 10^{-3} = 0.25 \times 8.314 \times T$
$\Rightarrow \quad T = 1654.7 \approx 1655$ K

Q.3 When the excited electron of a H atom from n = 5 drops to the ground state, the maximum number of emission lines observed are _______________.

3. **(4)**
For single H– atom, maximum number of spectral lines = $(n - 1)$, n = orbit number of excited electron.

Q.4 While performing a thermodynamics experiment, a student made the following observation.
$HCl + NaOH \rightarrow NaCl + H_2O$ $\Delta H = -57.3$ kJ mol^{-1}
$CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$ $\Delta H = -55.3$ kJ mol^{-1}
The enthalpy of ionization of CH_3COOH as calculated by the student is __________kJ mol^{-1}. (nearest integer)

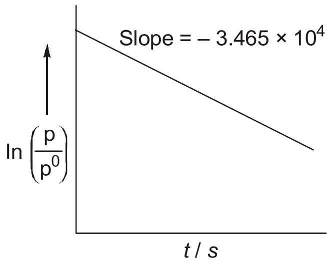
4. (2)

$\Delta H_{ionization}$ of $CH_3COOH = -55.3 - (-57.3)$

$= 2$ KJ / mole

Q.5 For the decomposition of azomethane,

$CH_3N_2CH_3(g) \rightarrow CH_3CH_3(g) + N_2(g)$, a first order reaction, the variation in partial pressure with time at 600 K is given as.



The half life of the reaction is __________ $\times 10^{-5} s$. [Neares integer]

5. (2)

For first order reaction.

$$In\left(\frac{P}{P_0}\right) = -k.t$$

$$K = 3.465 \times 104$$

$$t_{1/2} = \frac{0.693}{3.465 \times 10^4} = 2 \times 10^{-5} \text{ sec}$$

Q.6 The sum of number of lone pairs of electrons present on the central atoms of XeO_3, $XeOF_4$ and XeF_6, is ______________

6. (3)

$XeO_3 \Rightarrow$ Number of lone pair = 1

$XeOF_4 \Rightarrow$ Number of lone pair = 1

$XeF_6 \Rightarrow$ Number of lone pair = 1

Q.7 The spin-only magnetic moment value of M^{3+} ion (in gaseous state) from the pairs Cr^{3+} / Cr^{2+}, Mn^{3+} / Mn^{2+}, Fe^{3+} / Fe^{2+} and Co^{3+} / Co^{2+} that has negative standard electrode potential, is______ B.M [Nearest integer]

7. (4)

In given electrodes, only $E^0_{Cr^{3+}/Cr^{2+}}$ is negative

$Cr(24) - [Ar]\ 4s^1\ 3d^5 4p^0$

$Cr^{3+} - [Ar]\ 3d^3\ 4s^0 4p^0$

Number of unpaired electrons = 3

$M = \sqrt{n(n+2)}BM = \sqrt{3(3+2)}BM = 3.97$ BM ≈ 04 BM

Q.8 A sample of 4.5 mg of an unknown monohydric alcohol, R-OH was added to methylmagnesium iodide. A gas is evolved and is collected and its volume measured to be 3.1 mL. The molecular weigth of the unknown alcohol is _____________ g / mol

[Nearest integer]

8. (33)

(Calculation is done considering STP condition)

$ROH + CH_3MgI \longrightarrow CH_4 + ROMgI$

No of moles ROH = no of moles of CH_4

$$= \frac{4.5 \times 10^{-3}}{M} = \frac{31}{22400} \Rightarrow 32.52 \approx 33 \text{ gm / mole}$$

Q.9 The separation of two coloured substances was done by paper chromatography. The distance travelled by solvent front, substance *A* and substance *B* from the base line are 3.25 cm, 2.08 cm and 1.05 cm, respectively. The ratio of R_f values of *A* to *B* is ________.

9. (2)

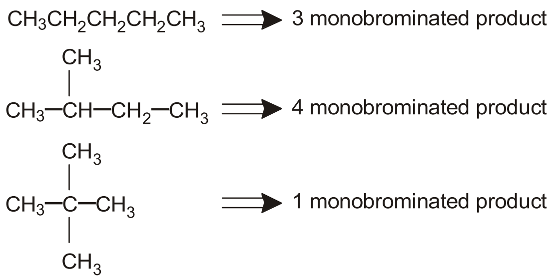
$$R_f^{\circ} = \frac{\text{dis tance travelled by the solute}}{\text{dis tance travelledby the solvent}}$$

$$(R_f)_A = \frac{2.08}{3.25} \qquad (R_f)_B = \frac{1.05}{3.25}$$

$$\frac{(R_f)_A}{(R_f)_B} = \frac{2}{1}$$

Q.10 The total number of monobromo derivatives formed by the alkanes with molecular formula C_5H_{12} is (excluding stereo isomers) ____________.

10. (8)



PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

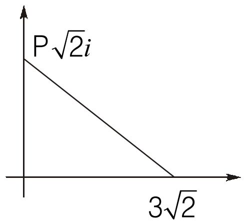
Q.1 For $z \in C$ if the minimum value of $\left(|z-3\sqrt{2}|+|z-p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of p is.............

(a) 3 (b) $\frac{7}{2}$

(c) 4 (d) $\frac{9}{2}$

1. **(c)**

$|z-3\sqrt{2}|+|z-p\sqrt{2}i|$ is minimum for $z, 3\sqrt{2}$ & $p\sqrt{2}i$ are collinear



$\Leftrightarrow \quad (3\sqrt{2})^2+(p\sqrt{2})^2 = (5\sqrt{2})^2$

$\Rightarrow \quad 18 + 2p^2 = 50$

$2p^2 = 32$

$p = \pm 4$

Q.2 The number of real values of λ, such that the system of linear equations

$2x - 3y + 5z = 9$

$x + 3y - z = -18$

$3x - y + (\lambda^2 - |\lambda|)z = 16$ has no solutions, is

(a) 0 (b) 1

(c) 2 (d) 4

2. **(c)**

System of equation can be written as

$$\begin{pmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2-|\lambda| \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ -18 \\ 16 \end{pmatrix}$$

$R_1 - 2R_2,\ R_3 - 3R_2$

$$\begin{pmatrix} 0 & -9 & 7 \\ 1 & 3 & -1 \\ 0 & -10 & \lambda^2-|\lambda|+3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 45 \\ -18 \\ 70 \end{pmatrix}$$

$\frac{9R_3}{10}$

$$\begin{pmatrix} 0 & -9 & 7 \\ 1 & 3 & -1 \\ 0 & -9 & \frac{9}{10}\left(\lambda^2-|\lambda|+3\right) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 45 \\ -18 \\ 63 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -9 & 7 \\ 1 & 3 & -1 \\ 0 & 0 & \frac{9}{10}\left(\lambda^2 - |\lambda| + 3\right) - 7 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 45 \\ -18 \\ 18 \end{pmatrix}$$

Q.3 The number of bijective functions $f : \{1, 3, 5, 7, ..., 99\} \to \{2, 4, 6, 8, ..., 100\}$, such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq .... \geq f(99)$, is.............. .

(a) ${}^{50}P_{17}$

(b) ${}^{50}P_{33}$

(c) $33! \times 17!$

(d) $\frac{50!}{2}$

3. (b)

$f : \{1, 3, 5, 7, ...,99\} \to \{2, 4, 6, 8, ...100\}$

$$f(3) \geq f(9) \geq f(15)..... \geq f(99)$$

$$3 + (n-1)6 = 99 \Rightarrow n = 17$$

$$\text{cases } f(3) > f(9) > f(15)......> f(99)$$

$\therefore$ from the set $\{2, 4, 6, ..., 100\}$

17 distinct numbers can be selected in ${}^{50}C_{17}$ ways again remaining $\{1, 5, 7, 11, ...\}$ can map in 33! ways

$\therefore$ total number of such required functions

$$= {}^{50}C_{17} \times 33!$$

$$= \frac{50!}{33!17!} \times 33! = {}^{50}P_{33}$$

Q.4 The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is

(a) 1

(b) 4

(c) 6

(d) 8

4. (d)

$$11 \equiv 2 \bmod (9)$$

$$(11)^{1011} \equiv 2^{1011} \bmod (9)$$

$$\text{Again } 2^3 \equiv -1 \bmod (9)$$

$$\Rightarrow \quad (2^3)^{337} \equiv (-1)^{337} \bmod (9)$$

$$\Rightarrow \quad 2^{1011} \equiv -1 \bmod (9) \equiv \bmod (9) \text{ .... } (i)$$

$$1011 \equiv 3 \bmod (9)$$

$$(1011)^{11} \equiv 3^{11} \bmod (9) \equiv 0 \bmod (9)$$

$$\therefore \quad (11)^{1011} + (1011)^{11} \equiv 8 \bmod (9)$$

$$\therefore \quad \text{Remainder} = 8$$

Q.5 The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to

(a) $\frac{7}{87}$

(b) $\frac{7}{29}$

(c) $\frac{14}{87}$

(d) $\frac{21}{29}$

5. (b)

$$\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$$

$$= \frac{3}{4}\sum_{n=1}^{21}\frac{(4n+3)-(4n-1)}{(4n-1)(4n+3)}$$

$$= \frac{3}{4}\sum_{n=1}^{21}\left[\frac{1}{4n-1}-\frac{1}{4n+3}\right]$$

$$= \frac{3}{4}\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+\ldots+\left(\frac{1}{4n-1}-\frac{1}{4n+3}\right)\right]$$

$$= \frac{3}{4}\left[\frac{1}{3}-\frac{1}{4n+3}\right] = \frac{3}{4}\frac{4n+3-2}{3(4n+3)} = \frac{n}{4n+3}$$

for $n = 21$

$$S_{21} = \frac{21}{84+3} = \frac{21}{87} = \frac{7}{29}$$

Q.6 $\lim_{x\to\frac{\pi}{4}}\frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ is equal to

(a) 14 (b) 7

(c) $14\sqrt{2}$ (d) $7\sqrt{2}$

6. (a)

$$\lim_{x\to\frac{x}{4}}\frac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}\left(\frac{0}{0}form\right)$$

$$= \lim_{x\to\frac{x}{4}}\frac{-7(\cos x+\sin x)^6(-\sin x+\cos x)}{-2\sqrt{2}\cos 2x}$$

$$= \underset{x\to\frac{\pi}{4}}{Lt}\ 7\frac{(\cos x+\sin x)^5\left(\cos^2 x-\sin^2 x\right)}{2\sqrt{2}\cos 2x} = \frac{7(\sqrt{2})^5}{2\sqrt{2}} = 14$$

Q.7 $\lim_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+\ldots+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$ is equal to

(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) – 2

7. (c)

$$I = \lim_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+\ldots+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$$

Let $2^n = t$ and if $n\to\infty$ then $t\to\infty$

$$I = \lim_{n\to\infty}\frac{1}{t}\left(\sum_{r=1}^{t=1}\frac{1}{\sqrt{1+\frac{r}{t}}}\right)$$

$$I = \int_0^1 \frac{dx}{\sqrt{1-x}} = \int_0^1 \frac{dx}{\sqrt{x}} \int_a^0 f(x)dx = \int_0^a f(a-x)dx$$

$$= \left[2x^{\frac{1}{2}}\right]_0^1 = 2$$

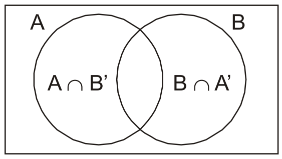
Q.8 If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$, then $P(A|B') + P(B|A')$ is equal to

(a) $\frac{3}{4}$ (b) $\frac{5}{8}$

(c) $\frac{5}{4}$ (d) $\frac{7}{8}$

8. (b)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$$\Rightarrow \quad \frac{1}{2} = \frac{1}{3} + \frac{1}{5} - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = \frac{1}{30}$$

$$P\left(\frac{A}{B'}\right) + P\left(\frac{B}{A'}\right)$$

$$= \frac{P(A \cap B')}{P(B')} + \frac{P(B \cap A')}{P(A')}$$

$$= \frac{P(A) - P(A \cap B)}{1 - P(B)} + \frac{P(B) - P(A \cap B)}{1 - P(A)} = \frac{5}{8}$$

Q.9 Let $[t]$ denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101}\left([\sin(\pi x)] + e^{[\cos(2\pi x)]}\right) dx$ is equal to

(a) $\frac{52(1-e)}{e}$ (b) $\frac{52}{e}$

(c) $\frac{52(2+e)}{e}$ (d) $\frac{104}{e}$

9. (b)

$$I = \int_{-3}^{103}\left([\sin(\pi x)] + e^{[\cos(2\pi x)]}\right) dx$$

$[\sin \pi x]$ is periodic with period 2 and

$e^{[\cos 2\pi x]}$ is periodic with period 1.

So,

$$I = 52\int_0^2\left([\sin(\pi x)] + e^{[\cos 2\pi x]}\right) dx$$

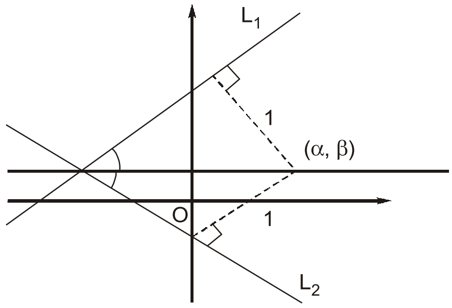
$$= 52\left\{\int_1^2 -1dx + \int_{1/4}^{3/4} e^{-1}dx + \int_{5/4}^{7/4} e^{-1}dx + \int_0^{1/4} e^0 dx + \int_{3/4}^{5/4} e^0 dx + \int_{7/4}^{2} e^0 dx\right\}$$

$$= \frac{52}{e}$$

Q.10 Let the point $P(\alpha, \beta)$ be at a unit distance from each of the two lines $L_1 : 3x - 4y + 12 = 0$, and $L_2 : 8x - 6y + 11 = 0$. If P lies below L_1 and above L_2, then $100(\alpha + \beta)$ is equal to

(a) – 14 (b) 42
(c) – 22 (d) 14

10. (d)



$$L_1 : 3x - 4y + 12 = 0$$
$$L_1 : 8x + 6y + 11 = 0$$

(α, β) lies on that angle which contain origin

∴ Equation of angle bisector of that angle which conta in origin is

$$\frac{3x - 4y + 12}{5} = \frac{8x + 6y + 11}{10}$$

$$\Rightarrow \quad 2x + 14y - 13 = 0$$

(α, β) lies on it

$$\Rightarrow \quad 2\alpha + 14\beta - 13 = 0 \quad ...(i)$$

$$\text{Again } \frac{3\alpha - 4\beta + 12}{5} = 1$$

$$\Rightarrow \quad 3\alpha - 4\beta + 7 = 0 \quad ...(ii)$$

Solving (*i*) and (*ii*)

$$\alpha = -\frac{23}{25} \ \& \ \beta = \frac{53}{50}$$

$$\therefore \quad \alpha + \beta = \frac{7}{50}$$

$$\Rightarrow \quad 100(\alpha + \beta) = 14$$

Q.11 Let a smooth curve $y = f(x)$ be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the points (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to

(a) $2 \log_e 2$ (b) 4
(c) 1 (d) $4 \log_e 2$

11. (b)

Slope of any point $P(x, y)$ to $y = f(x)$ is $\frac{dy}{dx} = -k\frac{y}{x}$

$$\Rightarrow \quad \frac{dy}{y} + k\frac{dx}{x} = 0$$

Solving the equation the curve is $x^k y = c$

It passes (1, 2) $\Rightarrow c = 2 \Rightarrow x^k y = 2$ again it passes (8, 1) $\Rightarrow 8^k = 2 \Rightarrow k = \frac{1}{3}$

∴ the equation of curve is $x^{1/3} y = 2$ ...(*i*)

$$\therefore \quad \left|y\left(\frac{1}{8}\right)\right| = \left|\frac{2}{\left(\frac{1}{8}\right)^{1/3}}\right| = 4$$

Q.12 If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the the x-axis and the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the y-axis then the eccentricity of the ellipse is

(a) $\frac{5}{7}$ (b) $\frac{2\sqrt{6}}{7}$

(c) $\frac{3}{7}$ (d) $\frac{2\sqrt{5}}{7}$

12. (a)

Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the points (7, 0) and $(0, -2\sqrt{6})$

$\therefore \quad a^2 = 49$ and $b^2 = 24$

$\therefore \quad e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{24}{49}} = \frac{5}{7}$

Q.13 The tangents at the points $A(1, 3)$ and $B(1, -1)$ on the parabola $y^2 - 2x - 2y = 1$ meet at the point P. Then the area (in unit^2) of the triangle PAB is :

(a) 4 (b) 6

(c) 7 (d) 8

13. (d)

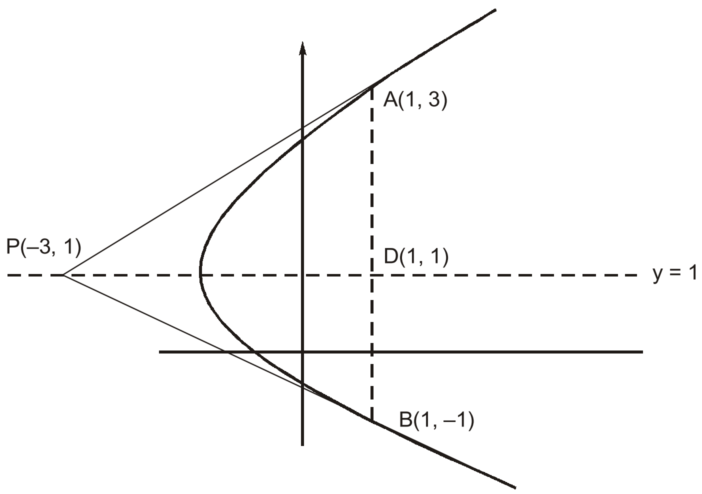
$$y^2 - 2x - 2y = 1$$

$\Rightarrow \quad (y-1)^2 = 2(x+1) \quad ...(i)$

Equation of tangent at A is $\quad 2x - y - 5 = 0 \quad ...(ii)$

D is mid point of AB solving (ii) with $y = 1$ $P(-3, 1)$

$\therefore \quad PD = 4, AD = 2$



Area of $\Delta APD = \frac{1}{2}(PD)(AD) = 4$

$\therefore$ Area of $\Delta APB = 8$ sq. units

Q.14 Let the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is :

(a) $\frac{32}{9}$ (b) $\frac{18}{5}$

(c) $\frac{27}{4}$ (d) $\frac{27}{10}$

14. (d)

Ellipse : $\frac{x^2}{16}+\frac{y^2}{7} = 1$

Eccentricity $= \sqrt{1-\frac{7}{10}} = \frac{3}{4}$

Foci $\equiv (\pm ae, 0) \equiv (\pm 3, 0)$

Hyperbola : $\frac{x^2}{\left(\frac{144}{25}\right)}-\frac{y^2}{\left(\frac{\alpha}{25}\right)} = 1$

Eccentricity $= \sqrt{1+\frac{\alpha}{144}} = \frac{1}{12}\sqrt{144+\alpha}$

Foci $\equiv (\pm ae, 0) \equiv \left(\pm\frac{12}{5}\cdot\frac{1}{12}\sqrt{144+\alpha}, 0\right)$

If foci coincide then

$$3 = \frac{1}{5}\sqrt{144+\alpha} \Rightarrow \alpha = 81$$

Hence, hyperbola is

$$\frac{x^2}{\left(\frac{12}{5}\right)^2}-\frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$$

Length of latus rectum

$$= 2\cdot\frac{\frac{81}{25}}{\frac{12}{5}} = \frac{27}{10}$$

Q.15 A plane E is perpendicular to the two planes $2x-2y+z=0$ and $x-y+2z=4$, and passes through the point $P(1,-1,1)$. If the distance of the plane E from the point $Q(a, a, 2)$ is $3\sqrt{2}$, then (PQ^2) is equal to

(a) 9 (b) 12

(c) 21 (d) 33

15. (c)

First plane, $P_1 = 2x-2y+z=0$

normal vector $\equiv n_1 = (2-2, 1)$

Second plane, $P_2 = x-y+2z=4$,

normal vector $\equiv n_2 = (1-1, 2)$

Plane perpendicular to P_1 and P_2 will have normal vector n_3 where

$$n_3 = (n_1 \times n_2)$$

Hence,

$$n_3 = (-3-3, 0)$$

Equation of plane E through $P(1,-1,1)$ and n_3 as normal vector

$$-3(x-1)-3(y+1) = 0$$

$\Rightarrow$ $$x + y = 0 \equiv E$$

Distance of $PQ(a,a,2)$ from $E = |2a/\sqrt{2}|$

as given,

$$\left|\frac{2n}{\sqrt{2}}\right| = 3\sqrt{2} \Rightarrow a = \pm 3$$

Hence, $Q \equiv (\pm 3, \pm 3, 2)$

Distance PQ

$$= \sqrt{21} \Rightarrow (PQ)^2 = 21$$

Q.16 The shortest distance between the liens $\frac{x+7}{-6} = \frac{y-6}{7} = z$ and $\frac{7-x}{2} = y-2 = z-6$ is

(a) $2\sqrt{29}$
(b) 1
(c) $\sqrt{\frac{37}{29}}$
(d) $\sqrt{\frac{29}{2}}$

16. (a)

$A(-7\hat{i}+6\hat{j})$ $\quad C(7\hat{i}+2\hat{j}+6\hat{k})$

$\vec{b} = -6\hat{i}+7\hat{j}+\hat{k}$ $\quad \vec{d} = -2\hat{i}+\hat{j}+\hat{k}$

$$\vec{b}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 7 & 1 \\ -2 & 1 & 1 \end{vmatrix} = 3\hat{i}+2\hat{j}+4\hat{k}$$

shortest distance between the lines

$$= \left|\frac{\overrightarrow{AC}\cdot(\vec{b}\times\vec{d})}{|\vec{b}\times\vec{d}|}\right| = \left|\frac{(14\hat{i}-4\hat{j}+6\hat{k})\cdot(3\hat{i}+2\hat{j}+4\hat{k})}{\sqrt{9+4+16}}\right|$$

$$= \left|\frac{42-8+24}{\sqrt{29}}\right| = \frac{58}{\sqrt{29}} = 2\sqrt{29}$$

Q.17 Let $\vec{a} = \hat{i}-\hat{j}+2\hat{k}$ and let $\vec{b}$ be a vector such that $\vec{a}\times\vec{b} = 2\hat{i}-\hat{k}$ and $\vec{a}\cdot\vec{b} = 3$. Then the projection of $\vec{b}$ on the vector $\vec{a}-\vec{b}$ is :

(a) $\frac{2}{\sqrt{21}}$
(b) $2\sqrt{\frac{3}{7}}$
(c) $\frac{2}{3}\sqrt{\frac{7}{3}}$
(d) $\frac{2}{3}$

17. (a)

$$\vec{a} = \hat{i}-\hat{j}+2\hat{k}$$

$$\vec{a}\times\vec{b} = 2\hat{i}-\hat{k}$$

$$\vec{a}\cdot\vec{b} = 3$$

$$|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = |\vec{a}|^2\cdot|\vec{b}|^2$$

$\Rightarrow$ $$5+9 = 6|\vec{b}|^2$$

$$|\vec{b}|^2 = \frac{7}{3}$$

$$|\vec{a}-\vec{b}| = \sqrt{|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}} = \sqrt{\frac{7}{3}}$$

$$\text{Projection of } |\vec{b}| \text{ on } \vec{a}-\vec{b} = \frac{\vec{b}\cdot(\vec{a}-\vec{b})}{|\vec{a}-\vec{b}|}$$

$$= \frac{\vec{b}\cdot\vec{a} - |\vec{b}|^2}{|\vec{a}-\vec{b}|} = \frac{3-\frac{7}{3}}{\sqrt{\frac{7}{3}}} = \frac{2}{\sqrt{21}}$$

Q.18 If the mean deviation about median for the numbers 3, 5, 7, 2k, 12, 16, 21, 24, arranged in the ascending order, is 6 then the median is

(a) 11.5
(b) 10.5
(c) 12
(d) 11

18. (d)

$$\text{Median} = \frac{2k+12}{2} = k+6$$

$$\text{Mean deviation} = \sum \frac{|x_i - M|}{n} = 6$$

$$\Rightarrow \frac{(k+3)+(k+1)+(k-1)+(6-k)+(6-k)+(10-k)+(15-k)+(18-k)}{8}$$

$$\therefore \frac{58-2k}{8} = 6$$

$$k = 5$$

$$\text{Median} = \frac{2\times 5+12}{2} = 11$$

Q.19 $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to :

(a) $\frac{3}{16}$
(b) $\frac{1}{16}$
(c) $\frac{1}{32}$
(d) $\frac{9}{32}$

19. (b)

$$2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$$

$$= 2\sin\left(\frac{11\pi-10\pi}{22}\right)\sin\left(\frac{11\pi-8\pi}{22}\right)\sin\left(\frac{11\pi-6\pi}{22}\right)\sin\left(\frac{11\pi-4\pi}{22}\right)\sin\left(\frac{11\pi-2\pi}{22}\right)$$

$$= \frac{2\sin\frac{32\pi}{11}}{2^5\sin\frac{\pi}{11}} = \frac{1}{16}$$

Q.20 Consider the following statements :

P : Ramu is intelligent.

Q : Ramu is rich.

R : Ramu is not honest.

The negation of the statement "Ramu is intelligent and honest if and only if Ramu is not rich" can be expressed as :

(a) $((P \wedge (\sim R)) \wedge Q) \wedge ((\sim Q) \wedge ((\sim P) \vee R))$

(b) $((P \wedge R) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \vee (\sim R)))$

(c) $((P \wedge R) \wedge Q) \wedge ((\sim Q) \wedge ((\sim P) \vee (\sim R)))$

(d) $((P \wedge (\sim R)) \wedge Q) \vee ((\sim Q) \wedge ((\sim P) \vee R))$

20. **(d)**

P : Ramu is intelligent.

Q : Ramu is rich.

R : Ramu is not honest.

Give statement, "Ramu is intelligent and honest if any only if Ramu is not rich"

$= (P \wedge \sim R) \Rightarrow \sim Q$

So, negation of the statement is

$\sim [(P \wedge \sim R) \Rightarrow \sim Q]$

$= \sim [\{\sim (P \wedge \sim R) \vee \sim Q\} \wedge \{Q \vee (P \wedge \sim R)\}]$

$= ((P \wedge \sim R) \wedge Q) \vee (\sim Q \wedge (\sim P \vee R))$

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q.1 Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Define $B = \{T \subseteq A :$ either $1 \notin T$ or $2 \in T\}$ and

$C = \{T \subseteq A : T$ the sum of all the elements of T is a prime number$\}$. Then the number of elements in the set $B \cup C$ is................

1. **(107)**

$\because \quad (B \cup C)' = B' \cap C'$

B' is a set containing sub sets of A containing element 1 and not containing 2.

And C' is a set containing subsets of A whose sum of elements is not prime.

So, we need to calculate number of subsets of

$\{3, 4, 5, 6, 7\}$ whose sum of elements plus 1 is composite.

Number of such 5 elements subset = 1

Number of such 4 elements subset = 3(except selecting 3 or 7)

Number of such 3 elements subset = 6(except selecting {3, 4, 5}, {3, 6, 7}, {4, 5, 7} or {5, 6, 7})

Number of such 2 elements subset = 7(except selecting {3, 7}, {4, 6}, {5, 7})

Number of such 1 elements subset = 3(except selecting {4} or {6})

Number of such 0 elements subset = 1

$n(B' \cap C') = 21 \quad \Rightarrow \quad n(B \cup C) = 2^7 - 21 = 107$

Q.2 Let $f(x)$ be a quadratic polynomial with leading coefficient 1 such that $f(0) = p$, $p \neq 0$, and $f(1) = \frac{1}{3}$. If the equations $f(x) = 0$ and $fo\ fo\ fo\ f(x) = 0$ have a common real root, then $f(-3)$ is equal to............ .

2. (25)

Let $f(x) = (x - \alpha)(x - \beta)$

It is given that $f(0) = p \Rightarrow \alpha\beta = p$ and $f(1) = \frac{1}{3} \Rightarrow (1-\alpha)(1-\beta) = \frac{1}{3}$

Now, let us assume that α is the common root of $f(x) = 0$ and fofofof $(x) = 0$

$$\text{fofofof}(x) = 0$$

$$\Rightarrow \quad \text{fofof}(0) = 0$$

$$\Rightarrow \quad \text{fof}(p) = 0$$

So, $f(p)$ is either α or β $(p - \alpha)(p - \beta) = \alpha$

$$(\alpha\beta - \alpha)(\alpha\beta - \beta) = \alpha \Rightarrow (\beta - 1)(\alpha - 1)\beta = 1$$

So, $\beta = 3\ (\because a \neq 0)$

$$(1 - \alpha)(1 - 3) = \frac{1}{3}$$

$$f(x) = \left(x - \frac{7}{6}\right)(x - 3)$$

$$f(-3) = \left(-3 - \frac{7}{6}\right)(3 - 3) = 25$$

Q.3 Let $A = \begin{vmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{vmatrix}$, $a, b \in R$. If for some $n \in N$, $A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$ then $n + a + b$ is equal to ....

3. (24)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = I + B$$

$$B^2 = \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$B^2 = 0$

$\therefore \quad A^n = (1 + B)^n = {}^nC_0 I + {}^nC_1 B + {}^nC_2 B^2 + + {}^nC_3 B^3 + \ldots$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & na & na \\ 0 & 0 & nb \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{n(n-1)ab}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & na & na + \frac{n(n-1)}{2}ab \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 48 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing we get $na = 48$, $nb = 96$ and $na + \frac{n(n-1)}{2}ab = 2160$

$$\Rightarrow \quad a = 4,\ n = 12 \text{ and } b = 8$$

$$\Rightarrow \quad n + a + b = 24$$

Q.4 The sum of the maximum and minimum values of the function $f(x) = |5x - 7| + [x^2 + 2x]$ in the interval $\left[\frac{5}{4}, 2\right]$, where $[t]$ is the greatest integer $\leq t$ is................

4. (15)

$$f(x) = |5x-7| + [x^2+2x]$$
$$= |5x-7| + [(x+1)^2] - 1$$

Critical points of $f(x) = \frac{7}{5}, \sqrt{5}-1, \sqrt{6}-1, \sqrt{7}-1, \sqrt{8}-1, 2$

$\therefore$ Maximum or minimum value of $f(x)$ occur at critical points or boundary points

$\therefore$ $$f\left(\frac{5}{4}\right) = \frac{3}{4}+4 = \frac{19}{4}$$
$$f\left(\frac{7}{4}\right) = 0+4 = 4$$

as both $|5x-7|$ and x^2+2x are increasing in nature after $x = 7/5$

$$f(2) = 3+8 = 11$$

$\therefore$ $$f\left(\frac{7}{5}\right)_{min} = 4 \text{ and } f(2)_{max} = 11$$

Sum is 4 + 11 = 15

Q.5 Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = \frac{4y^3+2yx^2}{3xy^2+x^3}, y(1) = 1$.

If for some $n \in \mathbb{N}$, $y(2) \in [n-1, n)$ then n is equal to ....

5. (3)

$$\frac{dy}{dx} = \frac{y}{x}\frac{(4y^2+2x^2)}{(3y^2+x^2)}$$

Put $$y = vx$$

$\Rightarrow$ $$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\Rightarrow$ $$v + x\frac{dv}{dx} = \frac{v(4v^2+2)}{(3v^2+1)}$$

$\Rightarrow$ $$x\frac{dx}{dx} = v\left(\frac{4v^2+2-3v^2-1}{3v^2+1}\right)$$

$\Rightarrow$ $$\int (3v^2+1)\frac{dv}{v^3+v} = \int \frac{dx}{x}$$

$\Rightarrow$ $$ln|v^3+v| = ln x + C$$

$\Rightarrow$ $$ln\left|\left(\frac{y}{x}\right)^3 + \left(\frac{y}{x}\right)\right| = ln x + C$$

$$\downarrow y(1) = 1$$

$\Rightarrow$ $$C = ln 2$$

For $y(2)$

$$ln\left(\frac{y^2}{8}+\frac{y}{2}\right) = 2ln2 \Rightarrow \frac{y^3}{8}+\frac{y}{2} = 4$$

$\Rightarrow$ $$[y(2)] = 2$$

$\Rightarrow$ $$n = 3$$

Q.6 Let f be a twice differentiable function on R. If $f'(0) = 4$ and $f(x) + \int_0^x (x-t)\, f'(t)\, dt = (e^{2x}+e^{-2x})\cos 2x + \frac{2}{a}x$, then $(2a+1)^5 a^2$ is equal to ....

6. (8)

$\because \quad f(x) + \int_0^x (x-t)\, f'(t)\, dt = (e^{2x} + e^{-2x})\cos 2x + \frac{2x}{a}$ ...(i)

Here $f(0) = 2$ ...(ii)

On differentiating equation (i) w.r.t. x we get :

$f'(x) + f\int_0^x f'(t)dt + xf'(x) - xf'(x)$

$= 2(e^{2x} - e^{-2x})\cos 2x - 2(e^{2x} + e^{-2x}) \sin 2x + \frac{2}{a}$

$\Rightarrow f(x) + f(x) - f(0) = 2(e^{2x} - e^{-2x})\cos 2x - 2(e^{2x} + e^{-2x}) \sin 2x + (2 / a)$

Replace x by 0 we get :

$\Rightarrow 4 = \frac{2}{a} \Rightarrow a = \frac{1}{2}$

$\therefore \quad (2a + 1)^5 \cdot a^2 = 2^5 \cdot \frac{1}{2^2} = 2^3 = 8$

Q.7 Let $a_n = \int_{-1}^{n}\left(1 + \frac{x}{2} + \frac{x^2}{3} + \ldots + \frac{x^{n-1}}{n}\right) dx$ for every $n \in N$. Then the sum of all the elements of the set $\{n \in N : a_n \in (2, 30)\}$ is ....

7. (5)

$\because \quad a_n = \int_{-1}^{n}\left(1 + \frac{x}{2} + \frac{x^2}{2} + \ldots + \frac{x^{n-1}}{n}\right) dx$

$= \left[x + \frac{x^2}{2^2} + \frac{x^3}{3^2} + \ldots + \frac{x^n}{n^2}\right]_{-1}^{n}$

$a_n = \frac{n+1}{1^2} + \frac{n^2-1}{2^2} + \frac{n^3+1}{3^2} + \frac{n^4-1}{4^2} + \ldots + \frac{n^n + (-1)^{n+1}}{n^2}$

Here $a_1 = 2,\ a_2 = \frac{2+1}{1} + \frac{2^2-1}{2} = 3 + \frac{3}{2} = \frac{9}{2}$

$a_3 = 4 + 2 + \frac{28}{9} = \frac{100}{9}$

$a_4 = 5 + \frac{15}{4} + \frac{65}{9} + \frac{255}{16} > 31$

$\therefore$ The required set is {2, 3}.

$\because \quad a_n \in (2, 30)$

$\therefore$ Sum of elements = 5.

Q.8 If the circles $x^2 + y^2 + 6x + 8y + 16 = 0$ and

$x^2 + y^2 + 2(3 - \sqrt{3})x + 2(4 - \sqrt{6})y = k + 6\sqrt{3} + 8\sqrt{6}$ $k > 0$, touch internally at the point $P(\alpha, \beta)$, then $(\alpha + \sqrt{3})^2 + (\beta + \sqrt{6})^2$ is equal to ....

8. (25)

The circle $x^2 + y^2 + 6x + 8y + 16 = 0$ has centre $(-3, -4)$ and radius 3 units.

The circle

$x^2 + y^2 + 2(3 - \sqrt{3})x + 2(4 - \sqrt{6})y = k + 6\sqrt{3} + 8\sqrt{6},\ k > 0$ has centre $\left(\sqrt{3} - 3, \sqrt{6} - 4\right)$

and radius $\sqrt{k + 34}$

$\because$ These two circles touch internally hence

$$\sqrt{3}+6 = \left|\sqrt{k+34}-3\right|$$

Here, $k = 2$ is only possible ($\because k > 0$)

Equation of common tangent to two circle is

$$2\sqrt{3}x+2\sqrt{6}y+16+6\sqrt{3}+8\sqrt{6}+k = 0$$

$\because$ $k = 2$ equation is

$$x+\sqrt{2}y+3+4\sqrt{2}+3\sqrt{3} = 0 \qquad ...(i)$$

$\because$ (α, β) are foot of perpendicular from $(-3, -4)$ to line (i) then

$$\frac{\alpha+3}{1}=\frac{\beta+4}{\sqrt{2}} = \frac{-(-3-4\sqrt{2}+3+4\sqrt{2}+3+\sqrt{3})}{1+2}$$

$\therefore$ $$\alpha + 3 = \frac{\beta+4}{\sqrt{2}} = -\sqrt{3}$$

$\Rightarrow$ $$(\alpha+\sqrt{3})^2 = 9 \text{ and } (\beta+\sqrt{6})^2 = 16$$

$\therefore$ $$(\alpha+\sqrt{3})^2+(\beta+\sqrt{6})^2 = 25$$

Q.9 Let the area enclosed by the x-axis, and the tangent and normal drawn to the curve $4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at the point $(-2, 3)$ be A. Then $8A$ is equal to..............

9. (170)

$4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ differentiating both sides we get

$12x^2 - 3y^2 - 6xyy' + 12x - 5y - 5xy = 16yy' + 9 = 0$

At the point $(-2, 3)$

$\Rightarrow$ $48 - 27 + 36y' - 24 - 15 + 10\,y' - 48\,y' + 9 = 0$

$\Rightarrow$ $2y = -9 \;\Rightarrow m_T = \frac{-9}{2} \,\&\, m_N = \frac{2}{9}$

$\therefore$ Area $= \frac{1}{2} \times$ Base $\times$ Height

$$A = \frac{1}{2}\times\left(\frac{-4}{3}+\frac{31}{2}\right)(3) = \frac{1}{2}\left(\frac{85}{0}\right)\cdot 3 = \frac{85}{4} = 8A = 170$$

Q.10 Let $x = \sin(2\tan^{-1}\alpha)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$. If $S = \{\alpha \in R : y^2 = 1 - x\}$, then $\sum_{\alpha\in S} 16\alpha^3$ is equal to ....

10. (130)

$x = \sin(2\tan^{-1}\alpha) = \frac{2\alpha}{1+\alpha^2}$ .... (i)

and

$$y^2 = 1 - x$$

$$y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right) = \sin\left(\sin^{-1}\frac{1}{\sqrt{5}}\right) = \frac{1}{\sqrt{5}}$$

Now,

$$y^2 = 1 - x$$

$$\frac{1}{5} = 1-\frac{2\alpha}{1+\alpha^2} \Rightarrow 1 + \alpha^2 = 5 + 5\alpha^2 - 10\alpha \Rightarrow 2\alpha^2 - 5\alpha + 2 = 0$$

$\therefore$ $$\alpha = 2, \frac{1}{2} \therefore \sum_{a\in S} 16\alpha^3 = 16\times 2^3 + 16\times\frac{1}{2^3} = 130$$

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