MADE EASY & NEXT IAS GROUP

PRESENT

MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE (MAIN) 2022

Test Date: 29th July 2022 (Second Shift)

PAPER-1

Questions with Solutions

Time : 3 Hours **Maximum Marks: 300**

Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

IMPORTANT INSTRUCTIONS:

1. The test is of 3 hours duration.

2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.

3. This question paper contains **Three Parts**. **Part-A** is *Physics*, **Part-B** is *Chemistry* and Part-**C** is *Mathematics*. Each part has only two sections: Section-**A** and **Section-B**.

4. **Section – A :** Attempt all questions.

5. **Section – B :** Do any 5 questions out of 10 Questions.

6. **Section-A (01 – 20)** contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and –1 mark for wrong answer.

7. **Section-B (1 – 10)** contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

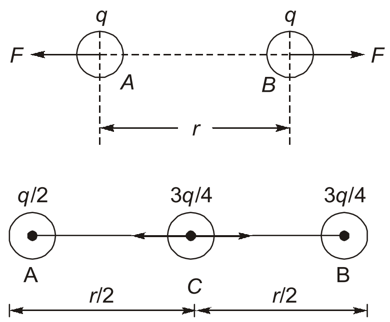
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q.1 Two identical metallic spheres *A* and *B* when placed at certain distance in air repel each other with force of *F*. Another identical uncharged sphere *C* is first placed in contact with *A* and then in contact with *B* and finally placed at midpoint between sphere *A* and *B*. The force experienced by sphere *C* will be :

(a) $3F/2$ (b) $3F/4$

(c) F (d) $2F$

1. (b)



For sphere '*C*' → after contacting with '*A*'. $q_A = q_C = \frac{q}{2}$.

offer contacting with '*B*'. $q_B = q_C = \frac{3q}{4}$

$$F_{net} = |F_1 - F_2|$$

$$F_2 = \frac{K \cdot 3q^2 \times 4}{r^2 8} = \frac{3}{2}\frac{kq^2}{r^2}$$

$$F_1 = \frac{9kq^2 \times 4}{16r^2} = \frac{9kq^2}{4r^2} = \frac{9}{4}F$$

$$\therefore \quad F_{net} = \left|\frac{9}{4}F - \frac{3}{2}F\right| = \frac{9-6}{4}F = \frac{3}{4}F$$

Q.2 Match List I with List II.

	List I		List II
A.	Torque	I.	Nms^{-1}
B.	Stress	II.	$J\ kg^{-1}$
C.	Latent Heat	III.	Nm
D.	Power	IV.	Nm^{-2}

Choose the correct answer from the options given below :

(a) A-III, B-II, C-I, D-IV (b) A-III, B-IV, C-II, D-I

(c) A-IV, B-I, C-III, D-II (d) A-II, B-III, C-I, D-IV

2. (b)

A. Torque → (iii) Nm

B. Stress → (iv) Nm^{-2}

C. Latent Heat → (ii) $J\ kg^{-1}$

D. Power → (i) $Nm\ S^{-1}$

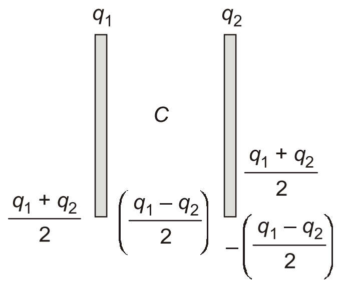
Q.3 Two identical thin metal plates has charge q_1 and q_2 respectively such that $q_1 > q_2$. The plates were brought close to each other to from a parallel plate capacitor of capacitance *C*. The potential difference between them is :

(a) $\frac{(q_1+q_2)}{C}$ (b) $\frac{(q_1-q_2)}{C}$

(c) $\left(\frac{q_1-q_2}{2C}\right)$ (d) $\frac{2(q_1-q_2)}{C}$

3. **(c)**

$$V = \frac{q_{(\text{capacitance})}}{C} = \frac{q_1-q_2}{2c}$$



Q.4 Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Alloys such as constantan and manganin are used in making standard resistance coils.

Reason R : Constantan and maganin have very small value of temperature coefficient of resistance.

In the light of the above statements, choose the correct answer from the options given below :

(a) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(b) Both **A** and **R** are true but **R** is Not the correct explanation of **A**.

(c) **A** is true but **R** is false.

(d) **A** is false but **R** is true.

4. **(a)**

Based on theoretical data.

Q.5 A 1 m long wire is broken into two unequal parts X and Y. The X part of the wire is stretched into another wire W. Length of W is twice the length of X and the resistance of W is twice that of Y. Find the ratio of length of X and Y.

(a) 1 : 4 (b) 1 : 2

(c) 4 : 1 (d) 2 : 1

5. **(b)**

$$R_w = 2\,R_y$$

$$\Rightarrow \quad \rho\left(\frac{2x}{A/2}\right) = \frac{2\rho(1-x)}{A}$$

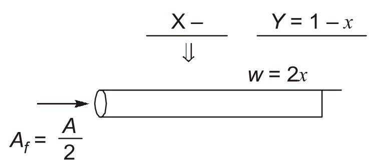
$$\Rightarrow \quad \frac{4\rho x}{A} = \frac{2\rho(1-x)}{A}$$

$$\Rightarrow \quad 4x = 2 - 2x$$

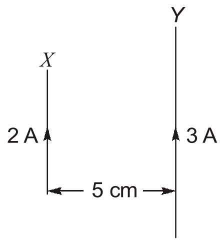
$$\Rightarrow \quad 6x = 2$$

$$x = \frac{2}{6} = \frac{1}{3}$$

$$\therefore \quad \frac{L_x}{L_y} = \frac{1/3}{1-1/3} = \frac{1}{3}\times\frac{3}{2} = \frac{1}{2}$$

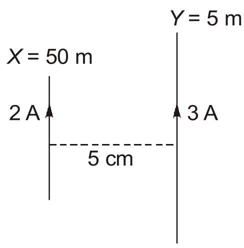


Q.6 A wire X of length 50 cm carrying a current of 2 A is placed parallel to a long wire Y of length 5 m. The wire Y current of 3 A. The distance between two wires is 5 cm and currents flow in the same direction. The force acting on the wire Y is



(a) 1.2×10^{-5} N directed towards wire X.
(b) 1.2×10^{-4} N directed away from wire X.
(c) 1.2×10^{-4} N directed towards wire X.
(d) 2.4×10^{-5} N directed towards wire X.

6. **(a)**



$$F_{x-y} = \frac{\mu_0 i_1 i_2}{2\pi r} \times (0.5)$$
$$= \frac{2\times10^{-7}\times6\times0.5}{0.05} = \frac{6}{5}\times10^{-5}N$$
$$= 1.2 \times 10^{-5}$$

Force on $Y = 1.2 \times 10^{-5}$
Force on $Y = 1.2 \times 10^{-5}$ towards x

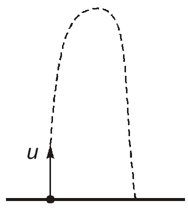
Q.7 A juggler throws ball vertically upwards with same initial velocity in air. When the first ball reaches its highest positions, he throws the next ball. Assuming the juggler throws n balls per second, the maximum height the balls can reach is

(a) $g/2n$ (B) g/n
(c) $2gn$ (D) $g/2n^2$

7. **(d)**

Time to reach at max height $t = \frac{u}{g}$ no. of balls throuwn in 1 sec = n.

So, time taken by each ball to reach maximum height, $= \frac{1}{n}$ sec



i.e. $$\frac{u}{g} = \frac{1}{n}$$

$\Rightarrow$ $$u = \frac{g}{n}$$

So, $$h_{max} = \frac{u^2}{2g} = \frac{g^2}{2gn^2} = \frac{g}{2n^2}$$

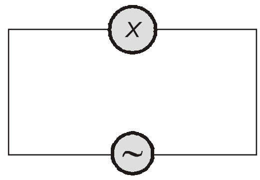
Q.8 A circuit element X when connected to an a.c. supply of peak voltage 100 V gives a peak current of 5 A which is in phase with the voltage. A second element Y when connected to the same a.c. supply also gives the same value of peak current which lags behind the voltage by $\frac{\pi}{2}$. If X and Y are connected in series to the same supply, what will be the rms value of the current in ampere?

(a) $\frac{10}{\sqrt{2}}$ (b) $\frac{5}{\sqrt{2}}$
(c) $5\sqrt{2}$ (d) $\frac{5}{2}$

8. (d)

$$v = 100 \sin \omega t$$

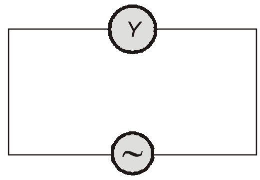
$$i_0 = \frac{100}{R_x}$$



$\Rightarrow$ $$R_x = 20\ \Omega$$

$$i = 5 \sin\left(\omega t - \pi/2\right)$$

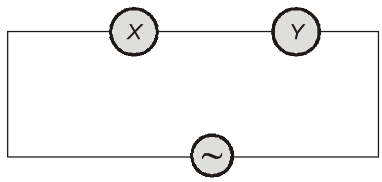
$$i = 100\ 5 \sin \omega t$$



$$5 = \frac{100}{R_y}$$

$\Rightarrow$ $$R_y = \frac{100}{5} = 20\ \Omega$$

When x and y both are connected in series:



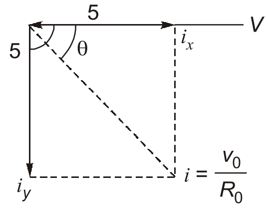
$$v = 100 \sin \omega t$$

$$\tan \theta = 1$$

$\Rightarrow$ $$\theta = 45^\circ$$

$$R_0 = \sqrt{R_x^2 + R_y^2} = 20\sqrt{2}\Omega$$

$\therefore$ $$\ell_0^\circ = \frac{v_0}{R_0} = \frac{100}{20\sqrt{2}} = \frac{5}{\sqrt{2}} A$$



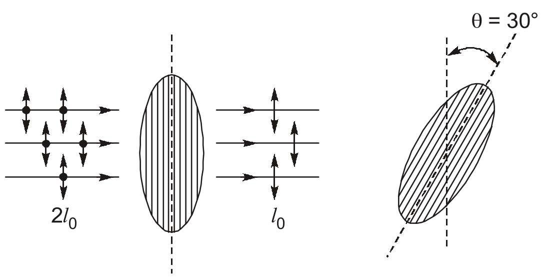
$\therefore$ $$\ell_{rms} = \frac{\ell_0}{\sqrt{2}} = \frac{5}{\sqrt{2}.\sqrt{2}} = \frac{5}{2} A$$

Q.9 An unpolarised light beam of intensity $2I_0$ is passed through a polaroid P and then through another polaroid Q which is oriented is such a way that its passing axis makes an angle of 30° relative to that of P. The intensity of the emergent light is

(a) $\frac{I_0}{4}$ (b) $\frac{I_0}{2}$

(c) $\frac{3I_0}{4}$ (d) $\frac{3I_0}{2}$

9. (c)



$$I = I_0 \cos^2 30^\circ$$

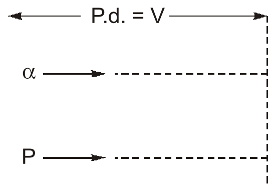
$$= I_0\left(\frac{\sqrt{3}}{2}\right) = \frac{3}{4} I_0$$

Q.10 An α particle and a proton are accelerated from rest through the same potential difference. The ratio of linear momenta acquired by above two particals will be :

(a) $\sqrt{2}:1$　(b) $2\sqrt{2}:1$

(c) $4\sqrt{2}:1$　(d) $8:1$

10. **(b)**



$$k_\alpha = 2\,eV$$

$\Rightarrow$ $$P_\alpha = \sqrt{2K_\alpha m_\alpha}$$

$\Rightarrow$ $$k_P = eV$$

$\Rightarrow$ $$P_P = \sqrt{2K_P m_P}$$

$\therefore$ $$\frac{P_\alpha}{P_P} = \sqrt{\frac{K_\alpha m_\alpha}{K_P m_P}} = \sqrt{\frac{2\times(4m_P)}{m_P}} = \sqrt{8}:1 = 2\sqrt{2}:1$$

Q.11 Read the following statements :

(i) Volume of the nucleus is directly proportional to the mass number.

(ii) Volume of the nucleus is independent of mass number.

(iii) Density of the nucleus is directly proportional to the mass number.

(iv) Density of nucleus is directly proportional to the cube root of the mass number.

(v) Density of the nucleus is independent of the mass number.

Choose the correct option from the following options.

(a) (i) and (iv) only　(b) (i) and (v) only.

(c) (ii) and (v) only　(d) (i) and (iii) only.

11. **(b)**

Radius, $$R = R_0 A^{1/3}$$

Q.12 An object of mass 1 kg is taken to a height from the surface of earth which is equal to three times that radius of earth. The gain in potential energy of the object will be

[If, $g = 10$ ms^{-2} and radius of earth = 6400 km]

(a) 48 MJ　(b) 24 MJ

(c) 36 MJ　(d) 12 MJ

12. **(a)**

$$m = 1\text{ kg}$$

$$\Delta U = \frac{-Gm_e m}{(R+h)} - \left(-\frac{Gm_e m}{R}\right)$$

$$\Delta U = \frac{GMm}{R} - \frac{GMm}{R+h}$$

$$= mg_0R - mg_0\frac{R}{\left(1+\frac{h}{R}\right)} = mg_0\left\{1-\frac{1}{\left(1+\frac{3R}{R}\right)}\right\}$$

$$= mg_0R\left(\frac{3}{4}\right) = \frac{3}{4}\times1\times10\times6400\text{ km}$$

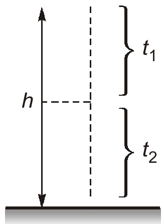
$$= 48000\times10^3\text{ J} = 48\text{ MJ}$$

Q.13 A ball is released from a height h. If t_1 and t_2 be the time required to complete first half and second half of the distance respectively. Then, choose the correct relation between t_1 and t_2.

(a) $t_1 = (\sqrt{2})t_2$ (b) $t_1 = (\sqrt{2}-1)t_2$

(c) $t_2 = (\sqrt{2}+1)t_1$ (d) $t_2 = (\sqrt{2}-1)t_1$

13. (d)



$$\frac{h}{2} = \frac{1}{2}gt_1^2$$

$$h = \frac{1}{2}g(t_1+t_2)^2$$

From (i) and (ii)

$$\frac{1}{2}gt_1^2 = \frac{g}{4}(t_1+t_2)^2$$

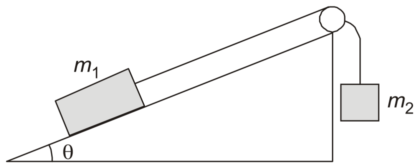
$\Rightarrow$ $$2t_1^2 = (t_1 + t_2)^2$$

$\Rightarrow$ $$\sqrt{2}t_1 = t_1 + t_2$$

$$t_1 = \frac{t_2}{\sqrt{2}-1} = (\sqrt{2}+1)t_2$$

$$t_2 = (\sqrt{2}-1)t_1$$

Q.14 Two bodies of masses m_1 = 5 kg and m_2 = 3 kg are connected by a light string going over a smooth light pulley on a smooth inclined plane as shown in the figure. The system is at rest. The force exerted by the inclined plane on the body of mass m_1 will be :
[Take g = 10 ms^{-2}]



(a) 30 N (b) 40 N

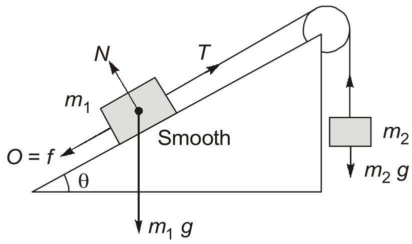
(c) 50 N (d) 60 N

14. (b)

$$m_1 = 5 \text{ kg}$$

$$m_2 = 3 \text{ kg}$$

As m_1 is in rest:



$$T = m_2g = 30$$

$$N = m_2g \cos\theta$$

$$N = 50 \cos\theta$$

$$T = m_2g \sin\theta$$

$\Rightarrow$ $$50 \sin\theta = 30$$

$\Rightarrow$ $$\sin\theta = \frac{3}{5}$$

$$\theta = 37^\circ$$

$\therefore$ $$N = 50\cos\theta = 50\cos 37^\circ = 50\times\frac{4}{5} = 40 \text{ N}$$

Q.15 If momentum of a body is increased by 20%, then its kinetic energy increases by

(a) 36% (b) 40%

(c) 44% (d) 48%

15. (c)

$$\frac{\Delta K}{K_i}\times 100 = \frac{K_f - K_i}{K_i}\times 100$$

$$= \frac{\frac{P_f^2}{2m} - \frac{P_i^2}{2m}}{\frac{P_i^2}{2m}}\times 100 = \frac{P_f^2 - P_i^2}{P_i^2}\times 100$$

$$= \frac{(1.2P_i)^2 - (P_i)^2}{P_i^2}\times 100 = \frac{1.44P_i^2 - P_i^2}{P_i^2}\times 100 = 44\%$$

Q.16 The torque of a force $5\hat{i}+3\hat{j}-7\hat{k}$ about the origin is τ. If the force acts on a particle whose position vector is $2\hat{i}+2\hat{j}+\hat{k}$, then the value of τ will be

(a) $11\hat{i}+19\hat{j}-4\hat{k}$ (b) $-11\hat{i}+19\hat{j}-16\hat{k}$

(c) $-17\hat{i}+19\hat{j}-4\hat{k}$ (d) $17\hat{i}+9\hat{j}+16\hat{k}$

16. (c)

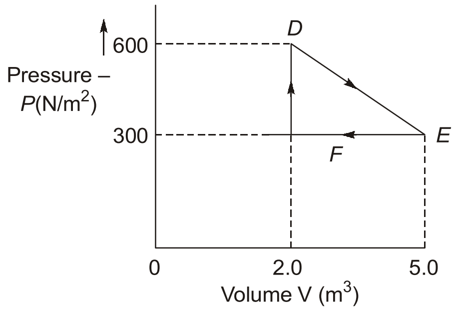
$$\vec{F} = 5\hat{i}+3\hat{j}-7\hat{k}$$

$$\vec{r} = 2\hat{i}+2\hat{j}+\hat{k}$$

$$\vec{\tau} = |\vec{r}\times\vec{F}| = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 2 & 1\\ 5 & 3 & -7\end{vmatrix} = \hat{i}\left(-k_1-3\right)-\hat{j}(-14-5)+\hat{k}(6-10)$$

$$= -17\hat{i}+19\hat{j}-4\hat{k}$$

Q.17 A thermodynamic system is taken from an original state *D* to an intermediate sate *E* by the linear process shown in the figure. Its volume is then reduced to the original volume from *E* to *F* by an isobaric process. The total work done by the gas from *D* to *E* to *F* will be



(a) –450 J (b) 450 J

(c) 900 J (d) 1350 J

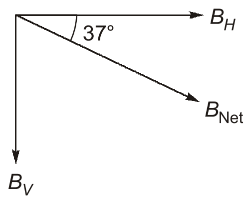
17. (b)

$$W_{total} = W_{D\to E} + W_{E\to F} = \text{Area of triangle } DEF$$
$$= \frac{1}{2}\times 3\times 300 = 450 \text{ J}$$

Q.18 The vertical component of the earth's magnetic field is 6×10^{-5} T at any place where the angle of dip is 37°. The earth's resultant magnetic field at that place will be (Given tan 37° = $\frac{3}{4}$)

(a) 8×10^{-5} T (b) 6×10^{-5} T
(c) 5×10^{-4} T (d) 1×10^{-4} T

18. (d)



$$B_v = 6 \times 10^{-5} \text{ T}$$
$$B_v = B_N \sin 37°$$
$$\therefore \quad B_N = \frac{B_v}{\sin 37°} = \frac{6\times 10^{-5}}{3/5} = 10^{-4} \text{ T}$$

Q.19 The root mean square speed of smoke particles of mass 5×10^{-17} kg in their Brownian motion in air at NTP is approximately. [Given : $k = 1.38 \times 10^{-23}$ JK^{-1}]

(a) 60 mm s^{-1} (b) 12 mm s^{-1}
(c) 15 mm s^{-1} (d) 36 mm s^{-1}

19. (c)

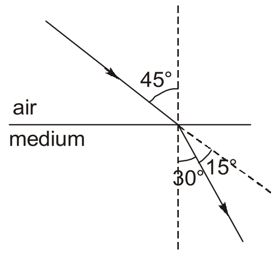
$$m = 5 \times 10^{-17} \text{ kg}$$
$$k = 1.38 \times 10^{-23} \text{ Jk}^{-1}$$
$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3NkT}{Nm}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\times 1.38\times 10^{-23}\times 300}{5\times 10^{-17}}}$$
$$= \sqrt{18\times 13.8\times 10^{-6}} = 15.7 \times 10^{-3} \text{ m/s}$$
$$= 15 \text{ mm/s}$$

Q.20 Light enters from air into a given medium at an angle of 45° with interface of the air-medium surface. After refraction, the light ray is deviated through an angle of 15° from its original direction. The refractive index of the medium is :

(a) 1.732 (b) 1.333
(c) 1.414 (d) 2.732

20. (c)

Using Snell's law:



$$\sin i = \mu \sin r$$
$$\Rightarrow \quad \sin 45° = \mu \sin 30°$$
$$\mu = \frac{\sin 45}{\sin 30} = \frac{1}{\sqrt{2}}\times 2 = \sqrt{2}$$

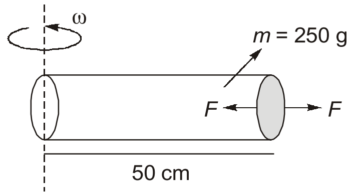
SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q.1 A tube of length 50 cm is filled completely with an incompressible liquid of mass 250 g closed both ends. The tube is then rotated in horizontal plane about one of its ends with a uniform angular velocity $x\sqrt{F}$ rad s^{-1}. If F be the force exertaed by the liquid at the other end then the value of x will be _____.

1. **(4)**



$$\omega = x\sqrt{F} \text{ rad/5}$$

As, $$F = m\omega^2 \frac{L}{2}$$

$\Rightarrow$ $$F = \frac{mL}{2}(x^2F)$$

$\Rightarrow$ $$x^2 = \frac{2}{mL} = \frac{2}{\frac{1}{4}\times\frac{1}{2}} = 16$$

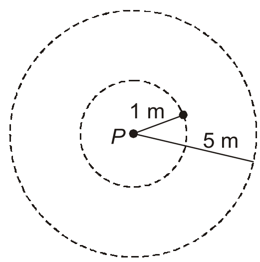
$\therefore$ $$x = 4$$

Q.2 Nearly 10% of the power of a 110 W light bulb is converted to visible radiation. The change in average intensities of visible radiation, at a distance of 1 m from the bulb to a distance of 5 m is a × 10^{-2} W/m^2. The value of 'a' will be ____.

2. **(84)**

$$P = 10\% \text{ of } 110 \text{ W} = 11 \text{ W} \qquad \text{(Visible)}$$

$$I_1 - I_2 = \frac{\rho}{4\pi}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) = \frac{11}{4\times\frac{22}{7}}\left(\frac{1}{1} - \frac{1}{25}\right)$$



$\Rightarrow$ $$\frac{7}{8}\times\left(\frac{24}{25}\right) = a \times 10^{-2} \text{ W/m}^2$$

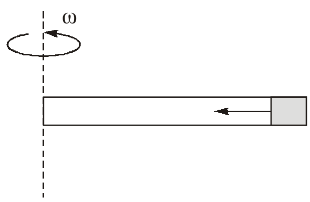
$\Rightarrow$ $$\frac{28\times 24}{8}\times 10^{-2} = a\times 10^{-2}$$

$\Rightarrow$ $$84 \times 10^{-2} = a \times 10^{-2}$$

$\Rightarrow$ $$a = 84$$

Q.3 A metal wire of length 0.5 m and cross-sectional area 10^{-4} m^2 has breaking stress 5 × 10^8 Nm^{-2}. A block of 10 kg is attached at one of the string and is rotating in a horizontal circle. The maximum linear velocity of block will be _____ ms^{-1}.

3. **(50)**



$$\ell = 0.5 \text{ m}$$

$$A = 10^{-4} \text{ m}^2$$

Breaking stress, $$\frac{F}{A} = 5 \times 10^8 \text{ N/m}^2$$

$\Rightarrow$ $$F = 5 \times 10^8 \times 10^{-4} \text{ N}$$

$$F = 5 \times 10^4 \text{ N}$$

$$T = \frac{mV^2}{\ell}$$

$$\Rightarrow \quad V^2_{(max)} = \frac{T\ell}{m} = \frac{5\times10^4\times0.5}{10}$$

$$V_m^2 = 2.5\times10^3$$

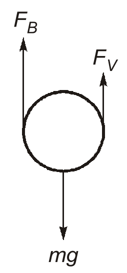
$$V_{max} = 50 \text{ m/s}$$

Q.4 The velocity of a small ball of mass 0.3 g and density 8 g/cc when dropped in a container filled with glycerine becomes constant after some time. If the density of glycerine is 1.3 g/cc, then the value of viscous force acting on the ball will be $x \times 10^{-4}$ N, value of x is _____. [use g = 10 m/s^2]

4. (25)

$$m = 0.3 \text{ g density}$$

$$\rho_{(ball)} = 80 \text{ g/cc}$$



$$\therefore \quad V = \frac{m}{\rho} = \frac{0.3}{8}\text{cc}$$

$$\rho_{glycerene} = 1.3 \text{ g/cc} = 1.3\times10^3 \text{ kg/m}^3$$

$$F_v + F_B = mg$$

$$\Rightarrow \quad F_v = mg - F_B$$

$$= 0.3\times10^{-3}\times10 - 1.3\times\frac{0.3}{8}\times10^{-6}\times10\times10^3$$

$$= 3\times10^{-3} - \frac{0.39}{8}\times10^{-2} = 3\times10^{-3} - 0.5\times10^{-3}$$

$$= 2.5\times10^{-3} \text{ N}$$

$$= 25\times10^{-4} \text{ N}$$

$$x = 25$$

Q.5 A modulating signal 2 sin$(6.28\times10^6)t$ is added to the carrier signal 4 sin$(12.56\times10^9)t$ for amplitude modulation. The combined signal is passed through a non-linear square law device. The output is then passed through a band pass filter. The bandwidth of the output signal of band pass filter will be ____ MHz.

5. (2)

Modulating signal $\rightarrow$ 2 sin $(6.28\times10^6)t$

Carrier signal $\rightarrow$ 4 sin $(12.56\times10^9)t$

Q.6 The speed of a transverse wave passing through a string of length 50 cm and mass 10 g is 60 ms^{-1}. The area of cross-section of the wire is 2.0 mm^2 and its Young's modulus is 1.2×10^{11} Nm^{-2}. The extension of the wire over its natural length due to its tension will be $x\times10^{-5}$ m. The value of x is _____.

6. (15)

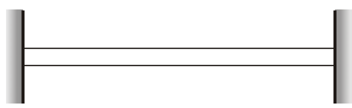
$$m = 10 \text{ g}$$

$$\ell = 50 \text{ cm}$$

$$A = 2 \text{ mm}^2$$

$$Y = 1.2\times10^{11} \text{ N/m}^2$$

$$\Delta x = x\times10^{-5} \text{ m}$$



As, $$\frac{T}{A} = Y\frac{\Delta x}{\ell}$$

$\Rightarrow \quad \Delta x = \frac{T\ell}{AY}$

$\Rightarrow \quad T\ell = V^2 m = \frac{V^2 m}{AY}$

$= \frac{3600 \times 10 \times 10^{-3}}{2 \times 10^{-6} \times 1.2 \times 10^{11}} = \frac{1800 \times 10^{-13} \times 10^{6}}{1.2}$

$= \frac{18}{12} \times 10^{-4} = \frac{3}{2} \times 10^{-4} = 15 \times 10^{-5}\text{m}$

So, $x = 15$

Q.7 The metallic bob of simple pendulum has the relative density 5. The time period of this pendulum is 10 s. If the metallic bob is immersed in water, then the new time period becomes $5\sqrt{x}\,s$. The value of x will be _____.

7. **(5)**

$$g_{eff} = g - \left(\frac{\rho_w}{\rho_b}\right) g$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

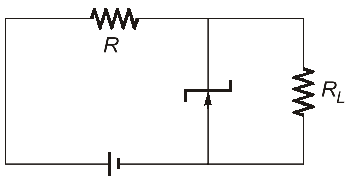
$$T' = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$

$$\frac{T'}{T} = \sqrt{\frac{g}{g_{eff}}} = \sqrt{\frac{5}{4}}$$

$\Rightarrow \quad T' = \sqrt{\frac{5}{4}}T = \sqrt{\frac{5}{4}} \times 10 = 5\sqrt{5}\,\text{sec}$

So, $x = 5$

Q.8 A 8 V Zener diode along with a series resistance R is connected across a 20 V supply (as shown in the figure). If the maximum Zener current is 25 mA, then the minimum value of R will be _____ Ω.



8. **(480)**

$i_{(zener\text{-}max)} = 25$ mA

$20 - i_{max} R - 8 = 0$

$i_{max} R = 12$

$\rightarrow$ At minimum zener current (μA):

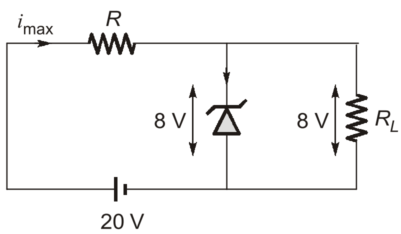
$\Rightarrow \quad 20 - i_{min} R - i_{min} R_L = 0$

$\Rightarrow \quad 20 - i_{min} R - 8 = 0$

$\frac{R}{R_L} = \frac{12}{8} = \frac{3}{2}$

$\ell_{min} R = 12$

$\ell_{min} R_L = 8$



At maximum zener current :

$$20 - i_{max} R - 8 = 0$$

$$i_L = 0 \{\text{as } i_z \text{ maximum} = 25 \text{ mA}\}$$

$$i_{max} R = 12 \text{ v}$$

$$25 \times 10^{-3} R = 12$$

$\Rightarrow$ $$R = \frac{12 \times 10^3}{25} = 12 \times 40 = 480\ \Omega$$

Q.9 Two radioactive materials A and B have decay constants 25λ and 16λ respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of B to that of A will be "e" after a time $\frac{1}{a\lambda}$. The value of a is _____.

9. **(9)**

$$\lambda_A = 25\lambda$$

$$\lambda_B = 16\lambda$$

At $t = 0$

$\Rightarrow$ $N_A = N_B = N_0$

After $t = \frac{1}{a\lambda}$: $$\frac{N_B}{N_A} = \frac{N_0 e^{-16\lambda t}}{N_0 e^{-25\lambda t}} = e^{(25\lambda - 16\lambda)t}$$

$\Rightarrow$ $$e = e^{\left(9\lambda \frac{1}{a\lambda}\right)}$$

$\Rightarrow$ $$\frac{9}{a} = 1$$

$\Rightarrow$ $$a = 9$$

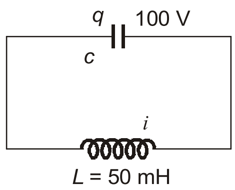
Q.10 A capacitor of capacitance 500 μF is charged completely using a dc supply of 100 V. It is now connected to an inductor of inductance 50 mH to form an LC circuit. The maximum current in LC circuit will be _____ A.

10. **(10)**

$$C = 500\,\mu\text{F}$$

$$V = 500 \text{ v}$$

$$L = 50 \text{ mH}$$



In this LC – oscillation

$$q = q_0 \cos \omega t$$

$$i = \frac{-dp}{dt} = q_0\, \omega \sin \omega t$$

$$\omega = \frac{1}{\sqrt{2c}} = \frac{1}{\sqrt{50 \times 10^{-3} \times 5 \times 10^{-4}}} = \frac{1000}{5} = 200$$

So, $$i_{max} = q_0\, \omega = 500 \times 10^{-6} \times 100 \times 200 = 10 \text{ A}$$

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q.1 Consider the reaction

$4HNO_3(l) + 3KCl(s) \rightarrow Cl_2(g) + NOCl(g) + 2H_2O(g) + 3KNO_3(s)$

The amount of HNO_3 required to produce 110.0 g of KNO_3 is

(Given: Atomic masses of H, O, N and K are 1, 16, 14 and 39, respectively)

(a) 32.2 g (b) 69.4 g

(c) 91.5 g (d) 162.5 g

1. **(c)**

$4HNO_3(l) + 3KCl(s) \rightarrow Cl_2(g) + NOCl(g) + 2H_2O(g) + 3KNO_3(s)$

$\downarrow$

4 moles of HNO_3 produced 3 mol of KNO_3

Here mole of produced $KNO_3 = \frac{110}{101}$

If 3 mol of KNO_3 produced by 4 moles of HNO_3

$\therefore$ 1 mole of KNO_3 produced by $\frac{4}{3}$ moles of HNO_3

and $\frac{110}{101}$ mole of KNO_3 produced by $\frac{4\times110}{3\times101}$ moles of HNO_3 = 1.45 mole of HNO_3

Hence mass of HNO_3 = mole × mol. wt = 145 × 63 = 91.48 ≈ 91.5 gm

Q.2 Given below are the quantum numbers for 4 electrons.

A. $n = 3, \ell = 2, m_\ell = 1, m_s = +1/2$

B. $n = 4, \ell = 1, m_\ell = 0, m_s = +1/2$

C. $n = 4, \ell = 2, m_\ell = -2, m_s = -1/2$

D. $n = 3, \ell = 1, m_\ell = -1, m_s = +1/2$

The correct order of increasing energy is

(a) D < B < A < C (b) D < A < B < C

(c) B < D < A < C (d) B < D < C < A

2. **(b)**

Sequence of sub-energy level decided by the rule of $(n + \ell)$

Q.3 $C(s) + O_2(g) \rightarrow CO_2(g) + 400\text{ kJ}$

$C(s) + \frac{1}{2}O_2(g) \rightarrow CO(g) + 100\text{ kJ}$

When coal of purity 60% is allowed to burn in presence of insufficient oxygen, 60% of carbon is converted into 'CO' and the remaining is converted into 'CO_2'.

The heat generated when 0.6 kg of coal is burnt is _____ .

(a) 1600 kJ (b) 3200 kJ

(c) 4400 kJ (d) 6600 kJ

3. (d)

Mass of pure carbon in coal = $0.6 \times 1000\text{gm} \times \frac{60}{100} = 360$ gm

Mass of carbon converted into CO = $\frac{60}{100} \times 360 = 216$ gm $\Rightarrow$ Mole = $\frac{216}{12} = 18$ mole of carbon

Mass of carbon converted into CO_2 = 360 – 216 = 144 gm $\Rightarrow$ Mole = $\frac{144}{12} = 12$ mole of carbon

$C_{(s)} + O_2(g) \rightarrow CO_2(g) + 400$ KJ

Mole = 12 for CO_2 production, Hence total energy produced = 400 × 12 = 4800 KJ

$C_{(s)} + \frac{1}{2}O_2(g) \longrightarrow CO_{(g)} + 100$ KJ

Mole = 18 for CO production, Hence energy produced = 100 × 18 = 1800 KJ

Total heat = 4800 + 1800 = 6600 KJ

Q.4 200mL of 0.01 M HCl is mixed with 400 mL of 0.01 M H_2SO_4. The pH of the mixture is _____.

Given: log2 = 0.30, log 3 = 0.48, log5 = 0.70, log 7 = 0.84, log 11 = 1.04

(a) 1.14 (b) 1.78

(c) 2.34 (d) 3.02

4. (b)

Total concentration of H^+ after the mixing of HCl and H_2SO_4:

$$[H^+] = \frac{(200 \times 0.01) + (400 \times 2 \times 0.01)}{600} = \frac{1}{60}$$

$\therefore$

$$pH = \log_{10}\left(\frac{1}{60}\right) = 1.78$$

Q.5 Given below are the critical temperature of some of the gases:

Gas	Critical temperature (K)
He	5.2
CH_4	190.0
CO_2	304.2
NH_3	405.5

The gas showing least adsorption on a definite amount of charcoal is

(a) He (b) CH_4

(c) CO_2 (d) NH_3

5. (a)

Higher adsorbtion of gas is corresponds to higher liquefaction and higher liquefaction is directly proportional to the higher critical temperature.

Q.6 In liquation process used for tin (Sn), the metal

(a) is reacted with acid.

(b) is dissolved in water.

(c) is brought to molten form which is made to flow on a slope.

(d) is fused with NaOH

6. (c)

Liquation refining process is applicable for the metal having low m.p, but containing impurities have higher m.p.

Q.7 Given below are two statements:

Statement I: Stannane is an example of a molecular hydride.

Statement II: Stannane is a planar molecule.

In the light of the above statement, choose the ***most appropriate*** answer from the options given below.

(a) Both statement I and Statement II are true.

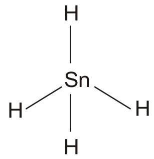
(b) Both statement I and Statement II are false.

(c) Statement I is true but Statement II is false.

(d) Statement I is false but Statement II is true.

7. (c)

Stannane is the tetrahedral shape of molecule.



Hybridisation is- sp^3, shape and structure are tetrahedral.

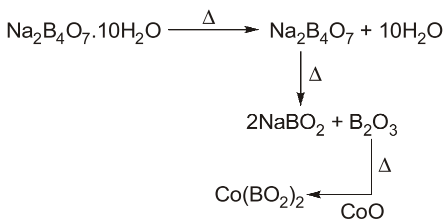
Q.8 Portland cement contains 'X' to enhance the setting time. What is 'X'?

(a) $CaSO_4 \bullet \frac{1}{2}H_2O$ (b) $CaSO_4 \bullet 2H_2O$

(c) $CaSO_4$ (d) $CaCO_3$

8. (b)

Here 'X' is the Gypsum ($CaSO_4.2H_2O$) which is used to enhance the setting of time.



Q.9 When borax is heated with CoO on a platinum loop, blue coloured bead formed is largely due to

(a) B_2O_3 (b) $Co(BO_2)_2$

(c) CoB_4O_7 (d) $Co[B_4O_5(OH)_4]$

9. (b)

Cobalt (II) meta borate is a blue colour of Bead.

Q.10 Which of the following 3d- metal ion will give the lowest enthalpy of hydration ($\Delta_{hyd}H$) when dissolved in water?

(a) Cr^{2+} (b) Mn^{2+}

(c) Fe^{2+} (d) Co^{2+}

10. (b)

Following are the ΔH^0 hydration of given ions

Cr^{2+} - 1926 KJ / mole

Mn^{2+} - 1860 KJ / mole

Fe^{2+} - 2000 KJ / mole

Co^{2+} - 2078 KJ / mole

Q.11 Octahedral complexes of copper (II) undergo structural distortion (Jahn-Teller). Which one of the given copper (II) complexes will show the maximum structural distortion? (en-ethylenediamine; $H_2N–CH_2–CH_2–NH_2$)

(a) $[Cu(H_2O)_6]SO_4$ (b) $[Cu(en)(H_2O)_4]SO_4$

(c) cis-$[Cu(en)_2Cl_2]$ (d) trans-$[Cu(en)_2Cl_2]$

11. (a)

According to the John-Tollen distortion, there is a unsymmetrical filling of eq set of Cu^{++} = ions but there is symmetrical filling of t_2g set of ions.

Q.12 Dinitrogen is a robust compound, but reacts at high altitudes to form oxides. The oxide of nitrogen that can damage plant leaves and retard photosynthesis is

(a) NO (b) NO_3^-

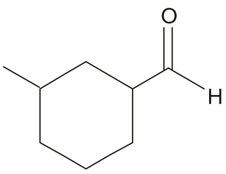
(c) NO_2 (d) NO_2^-

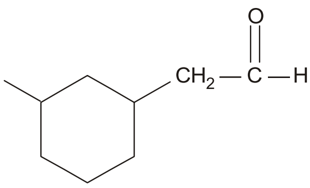
12. (c)

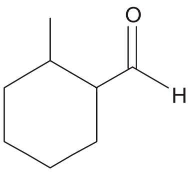
$N_2(g) + O_2(g) \rightarrow 2NO(g)$

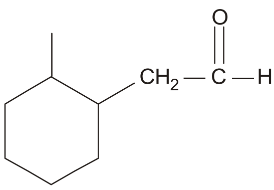
$2NO(g) + O_2(g) \rightarrow 2NO_2(g)$

Q.13 Correct structure of γ-methylcyclohexane carbaldehyde is

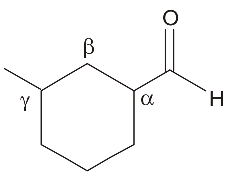
(a) 

(b) 

(c) 

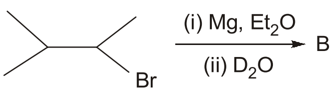
(d) 

13. (a)

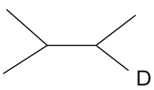


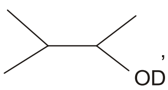
Q.14 Compound 'A' undergoes following sequence of reactions to give compound 'B'. The correct structure and chirality of compound 'B' is

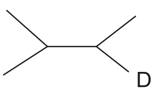
[where Et is $– C_2H_5$]

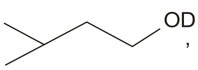


Compound 'A'

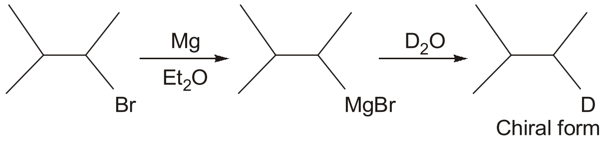
(a)  , Achiral

(b)  , chiral

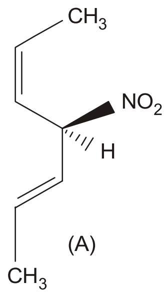
(c)  , chiral

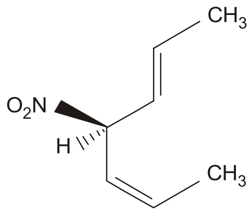
(d)  , Achiral

14. (c)



Q.15 Given below are two statements.

Statement I : The compound  is optically active.

Statement II :  is mirror image of above compound A.

In the light of the above statement, choose the most appropriate answer from the options given below.

(a) Both Statement I and Statement II are correct.
(b) Both Statement I and Statement II are incorrect.
(c) Statement I is correct but Statement II is incorrect.
(d) Statement I is incorrect but Statement II is correct.

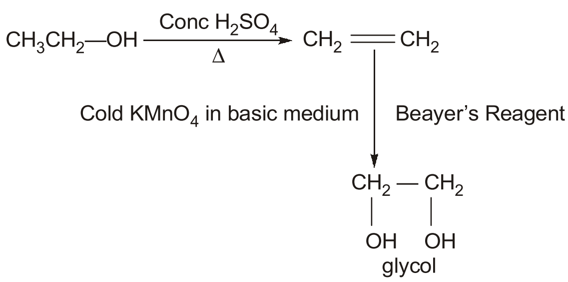
15. (c)

There is no mirror image of each other, having some configuration.

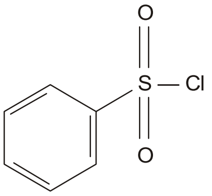
Q.16 When ethanol is heated with conc. H_2SO_4, a gas is produced. The compound formed, when this gas is treated with cold dilute aqueous solution of Baeyer's reagent, is

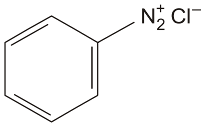
(a) formaldehyde (b) formic acid
(c) glycol (d) ethanoic acid

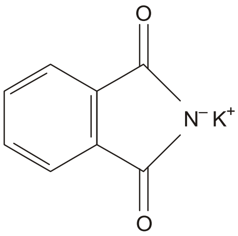
16. (c)

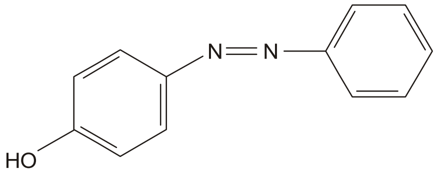


Q.17 The Hinsberg reagent is

(a) 

(b) 

(c) 

(d) 

17. (a)
Benzyl sulphonyl chloride is called Hinsberg's reagent.

Q.18 Which of the following is NOT a natural polymer?

(a) Protein (b) Starch
(c) Rubber (d) Rayon

18. (d)
Rayon is the semi synthetic polymer.

Q.19 Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A: Amylose is insoluble in water.
Reason R: Amylose is a long linear molecules with more than 200 glucose units.
In the light of the above statements, choose the correct answer from the options given below
(A) Both A and R are correct and R is the correct explanation of A
(B) Both A are R are correct but R is NOT the correct explanation of A
(C) A is correct but R is not correct
(D) A is not correct but R is correct

19. (d)
Amylose is the water soluble compound.

Q.20 A compound 'X' is a weak acid and it exhibits colour change at pH close to the equivalence point during neutralization of NaOH with CH_3COOH. Compound 'X' exists in ionized form in basic medium. The compound 'X' is

(a) methyl orange (b) methyl red
(b) phenolphthalein (d) erichrome Black T

20. (c)
Phenolphthalein is the indicator of weak acid which produces pink colour in basic medium.

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q.1 'x' g of molecular oxygen (O_2) is mixed with 200 g of neon (Ne). The total pressure of the non-reactive mixture of O_2 and Ne in the cylinder is 25 bar. The partial pressure of Ne is 20 bar at the same temperature and volume. The value of 'x' is
[Given: Molar mass of O_2 = 32 g mol^{-1}. Molar mass of Ne = 20 g mol^{-1}]

1. **(80)**

$$Po_2 = P_{Total} - P_{N_2}$$
$$= 25 - 20 = 5 \text{ bar}$$

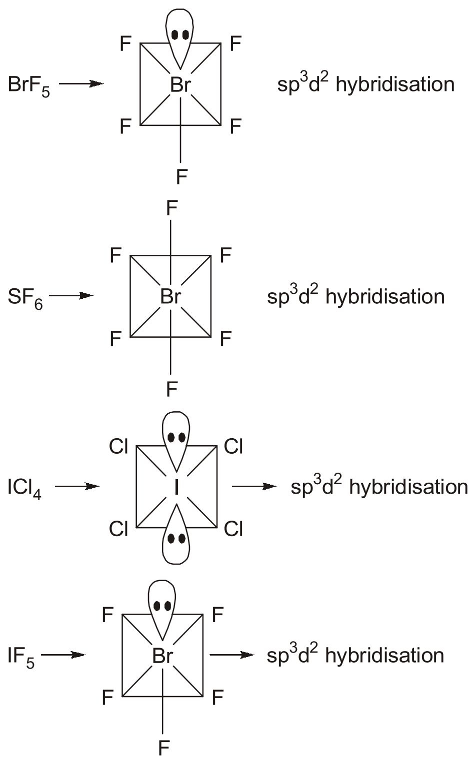
$$Po_2 = \frac{n_{O_2}}{n_{O_2} + n_{Ne}} \times P_{Total}$$

$$5 = \left(\frac{x/32}{x/32 + \frac{200}{20}}\right) \times 25$$

$\Rightarrow$ x = 80 gm

Q.2 Consider, PF_5, BrF_5, PCl_3, SF_6, $[ICl_4]^-$, ClF_3 and IF_5.
Amongst the above molecule(s)/ion(s), the number of molecule(s)/ion(s) having sp^3d^2 hybridisation is ____.

2. **(4)**
Following have the sp^3d^2 hybridisation



Q.3 1.80 g of solute A was dissolved in 62.5 cm^3 of ethanol and freezing point of the solution was found to be 155.1 K. The molar mass of solute A is _____ g mol^{-1}.
[Given: Feezing point of ethanol is 156.0 K. Density of ethanol is 0.80 g cm^{-3}. Freezing point depression constant of ethanol is 2.00 K kg mol-1]

3. (80)

$$\Delta T_f = K_f \times m$$

$$(156 - 155.1) = 2 \times \frac{1.8}{MW} \times \frac{1000}{(62.5 \times 0.8)}$$

$$0.9 = \frac{2 \times 1.8 \times 1000}{MW \times 50}$$

$$\therefore \quad MW = 80 \text{ gm/mol}$$

Q.4 For a cell, Cu(s) | Cu^{2+} (0.001M) || Ag^+(0.01M) | Ag(s)
The cell potential is found to be 0.43 V at 298 K. The magnitude of standard electrode potential for Cu^{2+}/Cu is _____. × 10^{-2} V.

$$\left[\text{Given} : E^{\Theta}_{Ag^+/Ag} = 0.80 \text{ V and } \frac{2.30\,RT}{F} = 0.06 \text{ V}\right]$$

4. (34)

$$Cu_{(s)} \longrightarrow Cu^{++} + 2e^- \quad \text{(Oxid. at cathode)}$$

$$\frac{2Ag^+ + 2e \longrightarrow 2Ag(s)}{Cu_{(s)} + 2Ag^+ \longrightarrow 2Ag_{(s)} + Cu^{++}} \quad \text{(Red. at anode)}$$

$$Q = \frac{[Cu^{++}]}{[Ag^+]^2}$$

and $n = 2$

$$0.43 = E^\circ_{cell} - \frac{0.06}{2} \log \frac{(0001)}{(001)^2}$$

$$\therefore \quad E^\circ_{cell} = 0.46$$

$$E^\circ_{cell} = E^\circ_{Ag^+/Ag} - E^\circ_{Cu^{++}/Cu}$$

$$0.46 = 0.80 - E^\circ_{Cu^{++}/Cu}$$

$$\therefore \quad E^\circ_{Cu^{++}/Cu} = 0.34 \text{ Volt} = 34 \times 10^{-2}$$

Q.5 Assuming 1μg of trace radioactive element X with a half life of 30 years is absorbed by a growing tree. The amount of X remaining in the tree after 100 years is _____ × 10^{-1} μg
[Given: In 10 = 2.303; log 2 = 0.30]

5. (1)

$$\lambda t = \ell n \frac{N_o}{N_t}$$

$$\frac{0.693}{t_{1/2}} \times t = \ell n \frac{1}{N_t}$$

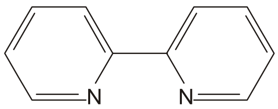
Or $$\frac{0.693 \times 100}{30} = \ell n \frac{1}{N_t}$$

$$\frac{1}{N_t} = 10$$

$\therefore$ $$N_t = \frac{1}{10} = 0.1$$

Or $$N_t = 1 \times 10^{-1}\ \mu g$$

Q.6 Sum of oxidation state (magnitude) and coordination number of cobalt in $Na[Co(bpy)Cl_4]$ is _____.

(Given: bpy = )

6. **(9)**

Oxidation state of Co = 3

And co-ordination No = 6

Sum = 3 + 6 = 9

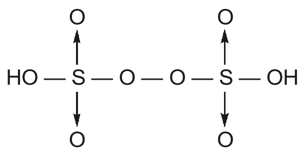
Q.7 Consider the following sulphur based oxoacids.

H_2SO_3, H_2SO_4, $H_2S_2O_8$ and $H_2S_2O_7$.

Amongst these oxoacids, the number of those with peroxo (O-O) bond is _____.

7. **(1)**

Only $H_2S_2O_8$ has per-oxo bond.



Q.8 A 1.84 mg sample of polyhydric alcoholic compound 'X' of molar mass 92.0 g / mol gave 1.344 mL of H_2 gas at STP. The number of alcoholic hydrogens present in compound 'X' is _____.

8. **(3)**

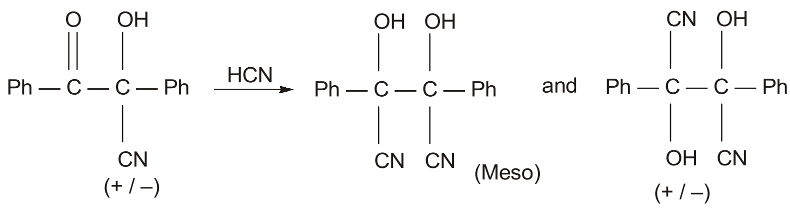
Mole of polyhydric alcohol = $\frac{1.84 \times 10^{-3}}{92} = 2.0 \times 10^{-5}$ mole.

Mole of H_2 gas produced = $\frac{1.344}{22400} = 6.0 \times 10^{-5}$ mole.

No of –OH gp present = $\frac{6.0 \times 10^{-5}}{2.0 \times 10^{-5}} = 3$

Q.9 The number of stereoisomers formed in a reaction of (±)Ph(C=O)C(OH)(CN) Ph with HCN is _____.
[where Ph is $-C_6H_5$]

9. **(3)**

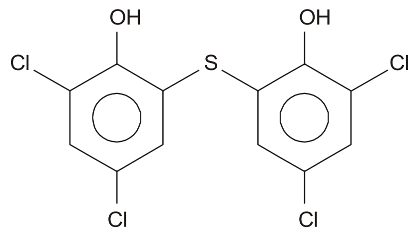


Total 3 streo- isomers

Q.10 The number of chlorine atoms in bithionol is _____.

10. (4)

Structure of Bithinol is;



PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q.1 If $z \neq 0$ be a complex number such that $\left|z-\frac{1}{z}\right|=2$, then the maximum value of z is :

(a) $\sqrt{2}$
(b) 1
(c) $\sqrt{2}-1$
(d) $\sqrt{2}+1$

1. (d)

$$\left|z-\frac{1}{z}\right| = 2$$

$$|z|_{max} = ?$$

$$\left|z-\frac{1}{2}\right| \geq \left||z|-\frac{1}{|z|}\right|$$

$$2 \geq \left|r-\frac{1}{r}\right|$$

$$0 \leq r^2+2r-1 \text{ and } r^2-2r-1 \leq 0$$

$$r = \frac{-2 \pm \sqrt{8}}{2} = -1 \pm \sqrt{2}$$

$$r = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

$r \geq \sqrt{2}-1$ and $0 \leq r \leq 1+\sqrt{2}$

$\sqrt{2}-1 \leq r \leq \sqrt{2}+1$

Q.2 Which of the following matrices can NOT be obtained from the matrix $\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$ by a single elementary row operation?

(a) $\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$
(b) $\begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}$
(c) $\begin{bmatrix}-1 & 2\\ -2 & 7\end{bmatrix}$
(d) $\begin{bmatrix}-1 & 2\\ -1 & 3\end{bmatrix}$

2. (c)

$A = \begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$ $\quad EA = \begin{bmatrix}a & c\\ b & d\end{bmatrix}\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$

$= \begin{bmatrix}-a+c & 2a-c\\ -b+d & 2b-d\end{bmatrix}$

For $a = c$ For $\left.\begin{matrix} -a+c=0 \\ 2a-c=1 \end{matrix}\right] \to a = 1, c = 1$ $E = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$d = b + 1,\ d = 1,\ b = 0$

$\left.\begin{matrix} b+d=1 \\ 2b-d=-1 \end{matrix}\right] \to b = 0, d = 1$ $R_1 \to R_1 \to R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

For $\left.\begin{matrix} -a+c=1 \\ 2a-c=1 \end{matrix}\right] \to a = 0, c = 1$

$\left.\begin{matrix} -b+d=-1 \\ 2b-d=2 \end{matrix}\right] \to b = 1, d = 0$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

For $\left.\begin{matrix} -a+c=-1 \\ 2a-c=2 \end{matrix}\right] \to a = 1, c = 0$

$\left.\begin{matrix} -b+d=-2 \\ 2b-d=7 \end{matrix}\right] \to b = 5, d = 3$ $\begin{bmatrix} 1 & 0 \\ 5 & 3 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$R_2 \to 5R_1 + 3R_2$

For $\left.\begin{matrix} -a+c=-1 \\ 2a-c=2 \end{matrix}\right] \to a = 1, c = 1$

$\left.\begin{matrix} -b+d=-1 \\ 2b-d=3 \end{matrix}\right] \to b = 2, d = 1$

(A) $R_1 \to R_1 \to R_2$
(B) $R_2 \to R_2 \to 2R_1$
(C) $R_2 \to 3R_2 \to 5R_1$

Q.3 If the system of equations

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$

$x + 2y + 3z = 14$

has infinitely many solutions, then $\alpha + \beta$ is equal to

(a) 8 (b) 36
(c) 44 (d) 48

3. **(c)**

$$\Delta = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 15 - 2\alpha + \alpha - 6 - 1 = 0$$

$$\alpha = 8$$

For $\alpha = 8$, equations are

$$x + y + 3 = 6$$
$$2x + 5y + 8z = \beta$$
$$x + 2y + 3z = 14$$
$$(2, 5, 8) = \ell(1, 1, 1) + m(1, 2, 3)$$

$$\left.\begin{matrix} 2 = \ell + m \\ 5 = \ell + 2m \end{matrix}\right] \to 3 = m, \ell = -1$$

$$8 = \ell + 3m$$
$$\beta = 6\ell + 14m$$
$$= -6 + 42 = 36$$
$$\alpha + \beta = 8 + 36 = 44$$

Q.4 Let the function $f(x)\begin{cases} \dfrac{\log_e(1+5x)-\log_e(1+\alpha x)}{x} & ; \text{ if } x \neq 0 \\ 10 & ; \text{ if } x = 0 \end{cases}$ be continuous at $x = 0$.

Then α is equal to

(a) 10 (b) –10

(c) 5 (d) –5

4. (d)

$$\lim_{x\to 0} \frac{\ell n \dfrac{1+5x}{1+\alpha x}}{x} = 10$$

$$\lim_{x\to 0} \ell n \left(\frac{1+(5-\alpha)x}{\dfrac{(5-\alpha)x}{1+\alpha x} \cdot \left(\dfrac{1+\alpha x}{5-\alpha}\right)} \right)$$

$$5 - \alpha = 10$$
$$\alpha = -5$$

Q.5 If $[t]$ denotes the greatest integer $\leq t$, then the value of $\int_0^1 \left[2x - \left|3x^2 - 5x + 2\right| + 1\right] dx$ is:

(a) $\dfrac{\sqrt{37}+\sqrt{13}-4}{6}$ (b) $\dfrac{\sqrt{37}-\sqrt{13}-4}{6}$

(c) $\dfrac{-\sqrt{37}-\sqrt{13}+4}{6}$ (d) $\dfrac{-\sqrt{37}+\sqrt{13}+4}{6}$

5. (a)

$$y = 2x - |3x^2 - 5x + 2|, \ 0 \leq x \leq 1$$

$$= \begin{cases} -3x^2 + 7x - 2, 0 \leq x \leq \dfrac{2}{3} \\ 3x^2 - 3x + 2, \dfrac{2}{3} < x \leq 1 \end{cases}$$

$$3x^2 - 5x + 2 = 0$$

$$x = \frac{+5 \pm \sqrt{25-24}}{6} = \frac{5+1}{6} = 1, \frac{2}{3}$$

$$3x^2 - 7x + 3 = 0$$

$$x = \frac{7 \pm \sqrt{49-36}}{6} = \frac{7 \pm \sqrt{13}}{6}$$

$-3x^2 + 7x - 2$

$$\frac{-7 \pm \sqrt{49-24}}{-6} = \frac{7+5}{6} = 2, \frac{1}{3}$$

$$3x^2 + 7x - 2 = -1$$

$\Rightarrow 3x^2 - 7x + 1 = 0$

$$x = \frac{7 \pm \sqrt{49-12}}{6} = \frac{7 \pm \sqrt{37}}{6}$$

$$I = \int_0^{\frac{7-\sqrt{37}}{6}} \begin{pmatrix} (-2) \\ +1 \end{pmatrix} dx + \int_{\frac{7-\sqrt{37}}{6}}^{\frac{1}{3}} \begin{pmatrix} (-1) \\ +1 \end{pmatrix} dx + \int_{-\frac{1}{3}}^{\frac{7-\sqrt{13}}{6}} \begin{pmatrix} 0 \\ +1 \end{pmatrix} dx + \int_{\frac{7-\sqrt{13}}{6}}^{\frac{2}{3}} (1+1)dx + \int_{\frac{2}{3}}^{1} (1+1)dx$$

$$= (-1)\left(\frac{7-\sqrt{37}}{6}\right) + 0 + \frac{7-\sqrt{13}}{6} - \frac{1}{3} + 2\left(\frac{2}{3} - \frac{7-\sqrt{13}}{6}\right) + 2\left(1 - \frac{2}{3}\right)$$

$$= \frac{-7}{6} + \frac{7}{6} - \frac{1}{3} + \frac{4}{3} - \frac{14}{6} + \frac{2}{3} + \frac{\sqrt{37}}{6} - \frac{\sqrt{13}}{6} + \frac{2\sqrt{13}}{6}$$

$$= \frac{\sqrt{37} + \sqrt{13} - 4}{6}$$

Q.6 Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1$, $\forall_n \geq 0$.

Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to

(a) 483
(b) 528
(c) 575
(d) 624

6. (b)

$a_0 = 0,\ a_1 = 0$

$a_{n+2} = 3a_{n+1} - 2a_n + 1$

$a_{25}\ a_{23} - 2a_{25}a_{22} - a_{23}a_{24} + 4a_{22}a_{24} = ?$

$a_2 = 3a_1 - 2a_0 + 1$

$a_3 = 3a_2 - 2a_1 + 1$

$a_4 = 3a_3 - 2a_2 + 1$

$a_5 = 3a_4 - 2a_3 + 1$

$a_{n+2} = 3a_{n+1} - 2a_n + 1$

$(+) \Rightarrow (a_2 + a_3 + a_4 + ... + a_{n+1} + a_{n+2})$

$= 3\,(a_1 + a_2 + a_3 + ... + a_{n+1})$

$-2(a_0 + a_1 + a_2 + ... + a_n) + (n + 1)$

$\Rightarrow\ a_{n+2} = 2(a_2 + a_3 + ... + a_n + a_{n+1}) - 2(a_1 + a_2 + ... + a_n) + n + 1$

$a_{n+2} = 2a_{n+1} + n + 1$

$a_{25}\ a_{23} - 2a_{25}\ a_{22} - a_{23}\ a_{24} + 4a_{22}\ a_{24}$

$= a_{25}\,(a_{23} - 2a_{22}) - 2a_{24}\,(a_{23} - 2a_{22})$

$= (a_{25} - 2a_{24})\,(a_{23} - 2a_{22})$

As $a_{n+2} = 2a_{n+1} + n + 1$

$\Rightarrow a_{n+2} - 2a_{n+1} = n + 1$

$\Rightarrow a_{n+2} - 2a_n = n$

$\Rightarrow 24 \times 22 = 528$

Q.7 $\sum_{r=1}^{20}(r^2+1)(r!)$ is equal to

(a) $22! - 21!$
(b) $22! - 2(21!)$
(c) $21! - 2(20!)$
(d) $21! - 20!$

7. (b)

$$\sum_{r=1}^{20}\left(r^2+1\right)r!$$

$$t_r = (r^2+1)r!$$
$$= r^2 r! + r!$$
$$= r(r+1-1)r! + r!$$
$$= r((r+1)! - r!) + r!$$
$$= r(r+1)! - r!\,(r-1)$$
$$= r(r+1)! - (r-1)r!$$
$$= V_r - V_{r-1}$$

$$\sum_{r=1}^{20}(V_r - V_{r-1})$$

$$= V_1 - V_0$$
$$+V_2 - V_1$$
$$+V_3 - V_2$$
$$+V_{20} - V_{19}$$
$$= V_{20} - V_0 = 20\,(21!) - 0$$
$$= (22-2)\,(21!) = 22! - 2(21!)$$

Q.8 For $I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x} dx$, if $I\left(\frac{\pi}{4}\right) = 2^{1011}$, then

(a) $3^{1010} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

(b) $3^{1010} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

(c) $3^{1011} I\left(\frac{\pi}{3}\right) - I\left(\frac{\pi}{6}\right) = 0$

(d) $3^{1011} I\left(\frac{\pi}{6}\right) - I\left(\frac{\pi}{3}\right) = 0$

8. (a)

$$I(x) = \int \frac{\sec^2 x - 2022}{\sin^{2022} x} dx$$

$$= \int \sin^{-2022} x \cdot \sec^2 x\, dx - \int 2022 \sin^{-2022} x\, dx$$

$$= \sin^{-2022} x \cdot \tan x - \int (-2022)\sin^{-2023} x \cdot \cos x \cdot \tan x\, dx - \int 2022 \sin^{-2022} x\, dx$$

$$= \tan x \cdot \sin^{-2022} x + 2022\int \sin^{-2022} - 2022\int \sin^{-2022} x\, dx$$

$$I(x) = \tan x\,.\,\sin^{-2022} x + c$$

Given, $I\left(\frac{\pi}{4}\right) = 2^{1011}$

$\Rightarrow$ $2^{1011} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^{2022}} + c \Rightarrow c = 0$

$$I(x) = \frac{\tan x}{\sin^{2022} x}, 1\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}} = \frac{2^{2022}}{(\sqrt{3})^{2021}}$$

Q.9 If the solution curve of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passes through the points (2, 1) and (k + 1, 2), $k > 0$, then

(a) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(b) $\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(c) $2\tan^{-1}\left(\frac{1}{k+1}\right)=\log_e\left(k^2+2k+2\right)$

(d) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(\frac{k^2+1}{k^2}\right)$

9. (a)

$$\frac{dy}{dx} = \frac{x+y-2}{x-y}$$

Let $x - 1 = X$, $y - 1 = Y$

then DE : $$\frac{dY}{dX} = \frac{X+Y}{X-Y} = \frac{1+\frac{Y}{X}}{1-\frac{Y}{X}}$$

Put $y = vx$

then $$\frac{dY}{dX} = V+X\frac{dV}{dX}$$

$$V+X\frac{dV}{dX} = \frac{1+V}{1-V}$$

$$X\frac{dV}{dX} = \frac{1+V}{1-V}-V = \frac{1+V-V+V^2}{1-V} = \frac{1+V^2}{1-V}$$

$$\frac{1-V}{1+V^2}dV = \frac{dX}{X}$$

$$\frac{V-1}{V^2+1}dV+\frac{dX}{X} = 0$$

$$\frac{1}{2}\ell n\left|V^2+1\right|-\tan^{-1}V+\ell n\,|X| = c$$

$$\ell n\sqrt{V^2+1}\cdot X-\tan^{-1}V = c$$

$$\ell n\left(\sqrt{1+\left(\frac{Y-1}{X-1}\right)^2}\cdot|X-1|\right)-\tan^{-1}\frac{Y-1}{X-1} = c$$

$$(2,1)\Rightarrow \ln(\sqrt{1+0}\cdot 1)-0 = c$$

$c = 0$

$$\ell n\sqrt{(X-1)^2+(Y-1)^2} = \tan^{-1}\frac{Y-1}{X-1}$$

point (k + 1, 2) ⇒ $$\ell n\sqrt{k^2+1} = \tan^{-1}\frac{1}{k}$$

⇒ $$\frac{1}{2}\ell n\left(k^2+1\right) = \tan^{-1}\frac{1}{k}$$

Q.10 Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{(x+3)}{x+1}, x>-1$, which passes through the point (0, 1). Then $y(1)$ is equal to:

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{5}{2}$ (d) $\frac{7}{2}$

10. (b)

$$\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y = \frac{x+3}{x+1}, x>-1$$

$$\text{IF} = e^{\int pdx} = \frac{(x+1)^2(x+2)}{x+3}$$

$$\int Pdx = \int\frac{2x^2+11x+13}{x^3+6x^2+11x+6}dn = \int\left(\frac{2}{x+1}+\frac{1}{x+2}-\frac{1}{x+3}\right)dx$$

$$= \ell n\left((x+1)^2\cdot(x+2)/(x+3)\right)$$

$$\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)} = \frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x+3}$$

$\Rightarrow$ $2x^2 + 11x + 13 = A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)$

$x = -1$

$\Rightarrow$ $4 = 2A \Rightarrow A = 2$

$x = -2$

$\Rightarrow$ $-1 = -B \Rightarrow B = 1$

$x_2 - 3 \Rightarrow -2 = 2c$

$c = -1$

$$y\cdot\frac{(x+1)^2(x+2)}{x+3} = \int\frac{x+3}{x+1}\cdot\frac{(x+1)^2(x+2)}{x+3}dx$$

$$= \int(x+1)(x+2)dx$$

$$= \int\left(x^2+3x+2\right)dx = \frac{x^3}{3}+\frac{3x^2}{2}+2x+c$$

$$(0,1)\Rightarrow 1\cdot\frac{2}{3} = c$$

$$x=1\Rightarrow y\cdot(3) = \frac{1}{3}+\frac{3}{2}+2+\frac{2}{3}=\frac{3}{2}+3=\frac{9}{2}$$

$$y = \frac{3}{2}$$

Q.11 Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2+11a+3\left(m_1^2+m_2^2\right)=220$.

If one vertex of the square is $(10(\cos\alpha-\sin\alpha), 10(\sin\alpha-\cos\alpha))$, where $\alpha\in\left(0,\frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos\alpha-\sin\alpha)x+(\sin\alpha+\cos\alpha)y=10$, then $72(\sin^4\alpha+\cos^4\alpha)+a^2-3a+13$ is equal to:

(a) 119 (b) 128

(c) 145 (d) 155

11. (b)

$m_1 m_2 = -1$, for square a, b, c, d let

$A\,(10\,(\cos\alpha - \sin\alpha), 10(\sin\alpha + \cos\alpha))$

Diagonal : $(\cos\alpha - \sin\alpha)\,x + (\sin\alpha + \cos\alpha)\,y = 10$

BD (diagonal)

Dist. Of BD from A is

$$\frac{|10(\cos\alpha - \sin\alpha)^2 + 10(\sin\alpha + \cos\alpha)^2 - 10|}{\sqrt{2}} = \frac{a}{\sqrt{2}}$$

$$\frac{10}{\sqrt{2}} = \frac{a}{\sqrt{2}} \Rightarrow a = 10$$

Also, $a^2 + 11a + 3\left(m_1^2 + m_2^2\right) = 220$

$$\Rightarrow 210 + 3\left(cm_1^2 + m_2^2\right) = 220$$

$$m_1^2 + m_2^2 = \frac{10}{3}$$

Also, $m_1\ m_2 = -1$

$$\to m^2 + \frac{1}{m^2} = \frac{10}{3}$$

or $-\sqrt{3}, \frac{1}{\sqrt{3}}$

$$m = \sqrt{3}, \frac{-1}{\sqrt{3}}$$

$$m^4 - \frac{10}{3}m^2 + 1 = 0 \Rightarrow m^2 = \frac{\frac{10}{3} \pm \sqrt{\frac{100}{9} - 4}}{2} - \frac{\frac{10}{3} \pm \frac{8}{3}}{2} = 3, \frac{1}{3}$$

$$m = \pm\sqrt{3}, \pm\frac{1}{\sqrt{3}}$$

Diagonal AC :

$(\sin\alpha + \cos\alpha)\,x - (\cos\alpha - \sin\alpha)y$

$= 10\cos 2\alpha - 10\cos 2\alpha = 0$

Slope of AC $= \dfrac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} = \dfrac{\tan\alpha + 1}{1 - \tan\alpha} = \tan\underbrace{\left(\alpha + \frac{\pi}{4}\right)}_{\downarrow\ 750}\alpha = 30°$

$$? = 72\left(\frac{1}{16} + \frac{9}{16}\right) + 100 - 30 + 13 = \frac{720}{16} + 83 = 128$$

Q.12 The number of elements in the set $S = \left\{x \in R : 2\cos\left(\frac{x^2 + x}{6}\right) = 4^x + 4^{-x}\right\}$ is :

(a) 1 (b) 3

(c) 0 (d) infinite

12. **(a)**

$$2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}$$

$-2 \le \text{LHS} \le 2$ LHS = 2 and RHS = 2 $\Rightarrow x = 0$ only $\rightarrow$ then LHS = 2 also
RHS ≥ 2

Q.13 Let $A(\alpha, -2)$, $B(\alpha, 6)$ and $C\left(\frac{\alpha}{4}, -2\right)$ be vertices of a ΔABC. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of ΔABC, then which of the following is NOT correct about ΔABC?

(a) area is 24 (b) perimeter is 25

(c) circumradius is 5 (d) inradius is 2

13. **(b)**

Circumcentre $(D) \equiv \left(5, \frac{\alpha}{4}\right)$

$$(5-\alpha)^2 + \left(\frac{\alpha}{4}+2\right)^2 = (5-\alpha)^2 + \left(\frac{\alpha}{4}-6\right)^2 \quad \text{... (i)}$$

$$\left(5-\frac{\alpha}{4}\right)^2 + \left(\frac{\alpha}{4}+2\right)^2 \quad \text{...(ii)}$$

$$\text{(i)} \Rightarrow \frac{\alpha}{4}+2 = \pm\left(\frac{\alpha}{4}-6\right)$$

(ii) $\Rightarrow 9 + 16 = 9 + 16$

$\oplus \rightarrow x$

$$(-) \rightarrow \frac{\alpha}{2} = 4 \Rightarrow \alpha = 8$$

ar $(ABC) = 24$

$2S = 24$

$$R = 5, r = \frac{\Delta}{s} = \frac{24}{12} = 2$$

Q.14 Let Q be the foot of perpendicular drawn from the point $P(1, 2, 3)$ to the plane $x + 2y + z = 14$. If R is a point on the plane such that $\angle PRQ = 60°$, then the area of $\angle PQR$ is equal to:

(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$

(c) $2\sqrt{3}$ (d) 3

14. **(b)**

$x + 2y + z = 14$

$$\perp^r \text{ line PQ} : \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{1} = t$$

$Q(1 + t, 2 + 2t, 3 + t)$

$x + 2y + z = 14 \Rightarrow 1 + t + 4 + 4t + 3 + t = 14 \Rightarrow 6t = 6$

$t = 1$

$\Rightarrow Q(2, 4, 4)$

$PQ = \sqrt{1+4+1} = \sqrt{6}$

$\tan 60^\circ = \dfrac{PQ}{QR} \Rightarrow QR = \dfrac{PQ}{\sqrt{3}} = \sqrt{2}$

$ar(PQR) = \dfrac{1}{\sqrt{2}}\sqrt{6}.\sqrt{2} = \sqrt{3}$

Q.15 If (2, 3, 9), (5, 2, 1), (1, λ, 8) and (λ, 2, 3) are coplanar, then the product of all possible values of λ is :

(a) $\dfrac{21}{2}$ (b) $\dfrac{59}{8}$

(c) $\dfrac{57}{8}$ (d) $\dfrac{95}{8}$

15. (d)

$A(2, 3, 9)$
$B(5, 2, 1)$
$C(1, \lambda, 8)$
$D(\lambda, 2, 3)$

$$\Delta = \begin{vmatrix} 3 & -1 & -8 \\ -4 & \lambda-2 & 7 \\ \lambda-2 & -1 & -6 \end{vmatrix}$$

$$\overrightarrow{AB} = (3, -1, -8)$$

$$\overrightarrow{AC} = (-4, \lambda-2, 7)$$

$$\overrightarrow{AD} = (\lambda-2, -1, -6)$$

$$\Delta = 3[-6\lambda + 12 + 7] - 1(7\lambda - 14 - 24) - 8(4 - (\lambda-2)^2)$$
$$= 57 - 18\lambda$$
$$\Rightarrow \quad = \lambda + 38 - 32 + 8(\lambda^2 - 4\lambda + 4)$$
$$= 95 - 57\lambda + 8\lambda^2$$
$$\Delta = 0$$

$$\Rightarrow \quad \lambda_1\lambda_2 = \frac{95}{8}$$

Q.16 Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is :

(a) $\dfrac{4}{9}$ (b) $\dfrac{5}{18}$

(c) $\dfrac{1}{6}$ (d) $\dfrac{3}{10}$

16. (b)

3R	2R
4B	5B
3W	2W
I	II

Let

E_1 : a red ball is transferred from I to II

E_2 : a black is transferred from I to II

E_3 : a white transferred from I to II

E : a black ball is drawn from 2nd bag after a ball from I to II was transferred.

$$P\left(\frac{E_1}{E}\right)=\frac{P(E_1\cap E)}{P(E)}$$

$P(E) = P(E_1 \cap E) + P(E_2 \cap E) + P(E_3 \cap E)$

$$=P(E_1)\cdot P\left(\frac{E}{E_1}\right)+\ldots+\ldots$$

$$=\frac{3}{10}\cdot\frac{5}{10}+\frac{4}{10}\cdot\frac{6}{10}+\frac{3}{10}\cdot\frac{5}{10}=\frac{54}{100}$$

$$P\left(\frac{E_1}{E}\right)=\frac{15/100}{54/100}=\frac{5}{18}$$

Q.17 Let $S = \{z = x + iy \,|z - 1 + i| \geq |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x, for which $w = 2x + iy \in S$ for some $y \in R$, is

(a) $\left(-\sqrt{2},\frac{1}{2\sqrt{2}}\right]$

(b) $\left(-\frac{1}{\sqrt{2}},\frac{1}{4}\right]$

(c) $\left(-\sqrt{2},\frac{1}{2}\right]$

(d) $\left(-\frac{1}{\sqrt{2}},\frac{1}{2\sqrt{2}}\right]$

17. (b)

$$|z - 1 + i| \geq |z|$$

$$|z + i| = |z - 1|$$

$$W = (2x, y) = (a, y)$$

Let S represent the line segment AB For 'B'

$$x^2 + y^2 = 4$$

$$x = -y$$

$\Rightarrow$ $$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$\Rightarrow B(-\sqrt{2},\sqrt{2})$

$A\left(\frac{1}{2},-\frac{1}{2}\right)$

$W(2x, y)$ lies on AB

$\Rightarrow$ $$-\sqrt{2} < 2x \leq \frac{1}{2}$$

$(|z| < 2)$

$\Rightarrow$ $$-\frac{1}{\sqrt{2}} < x \leq \frac{1}{4}$$

Q.18 Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})+(\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a})+(\vec{c}\times\vec{a})\cdot(\vec{a}\times\vec{b})=168$, then $|\vec{a}|+|\vec{b}|+|\vec{c}|$ is equal to:

(a) 10 (b) 14

(c) 16 (d) 18

18. (c)

Given : $\hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{c}=\hat{c}\cdot\hat{a} = \cos\theta$ (say)

$|\vec{a}||\vec{b}||\vec{c}| = 14$

$(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c})$

$= \vec{a}\cdot\vec{b}\times(\vec{b}\times\vec{c})$

$= \vec{a}\cdot[(\vec{b}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{b})\vec{c}]$

$= (\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c})-|\vec{b}|^2\,\vec{a}\cdot\vec{c}$

$= |\vec{a}||\vec{b}|^2|\vec{c}|\left(\cos^2\theta-\cos\theta\right)=14|\vec{b}|\left(\cos^2\theta-\cos\theta\right)$

Similarly, $(\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a})$

$= (\vec{b}\cdot\vec{c})(\vec{c}\cdot\vec{a})-|\vec{c}|^2(\vec{b}\cdot\vec{a})$

$= |\vec{b}||\vec{c}|^2|\vec{a}|\left(\cos^2\theta-\sin\theta\right)$

$= 14|\vec{c}|(\cos 2\theta-\sin\theta)$ & $(\vec{c}\times\vec{a})\cdot(\vec{a}\times\vec{b})$

$= (\vec{c}\cdot\vec{a})(\vec{a}\cdot\vec{b})-|\vec{a}|^2(\vec{c}\cdot\vec{b})$

$= |\vec{c}||\vec{a}|^2|\vec{b}|\left(\cos^2\theta-\sin\theta\right)$

$= 14|\vec{a}|\left(\cos^2\theta-\sin\theta\right)$

Given : $14\left(\cos^2\theta-\cos\theta\right)(|\vec{a}|+|\vec{b}|+|\vec{c}|)=168$

$$|\vec{a}|+|\vec{b}|+|\vec{c}| = \frac{12}{\cos^2\theta-\cos\theta}=\frac{12}{\frac{1}{4}-\left(-\frac{1}{2}\right)}=\frac{12}{\frac{3}{4}}=16$$

Given : $\vec{a}, \vec{b}, \vec{c}$ are coplanar and pair wise equal angle.

Q.19 The domain of the function $f(x)=\sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ is :

(a) $[1, \infty)$ (b) $[-1, 2]$

(c) $[-1, \infty)$ (d) $(-\infty, 2]$

19. (c)

$$-1 \le \frac{x^2-3x+2}{x^2+2x+7} \le 1$$

$$\Rightarrow \quad 0 \le \frac{2x^2-x+9}{x^2+2x+7} \ \& \ \frac{-5x-5}{x^2+2x+7} \le 0$$

$$\Rightarrow \quad x \in R \quad \text{and} \quad -1 \le x < \infty$$

Q.20 The statement $(p \Rightarrow q) \vee (p \Rightarrow r)$ is NOT equivalent to

(a) $(p \wedge (\sim r)) \Rightarrow q$

(b) $(\sim q) \Rightarrow ((\sim r) \vee p)$

(c) $p \Rightarrow (q \vee r)$

(d) $(p \wedge (\sim q)) \Rightarrow r$

20. (b)

(a) $\sim(p \wedge (\sim r)) \vee q = (\sim p) \vee (r) \vee q$

(b) $q \vee (\sim r \vee p)$

(c) $(\sim p) \vee (q \vee r)$

(d) $(\sim p \vee q) \vee r$

Using Venn diagram we get b as the correct option.

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

Q.1 The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. Then the number of trials in the binomial distribution is...........

1. (96)

$np + npq = 82.5 \Rightarrow np(1+q) = 82.5$

$np \cdot npq = 1350 \Rightarrow (np)^2 q = 1350$

$n = ?$

$$\Rightarrow \frac{(1+q)^2}{q} = \frac{82.5 \times 82.5}{1350}$$

$$q^2 + 2q + 1 = \frac{121}{24} q$$

$$\Rightarrow 24q^2 - 73q + 24 = 0$$

$$q = \frac{73 \pm \sqrt{73^2 - 4 \times 576}}{48} = \frac{73 \pm 55}{48} = \frac{128}{48}, \frac{18}{48} = \frac{8}{3}, \frac{3}{8}$$

$$p = \frac{5}{8}, q = \frac{3}{8}$$

$$n \cdot \frac{5}{8} \cdot \frac{11}{8} = \frac{165}{2}$$

$$n = 96$$

Q.2 Let $\alpha, \beta (\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in N$, then

$$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$$ is equal to ....

2. (16)

$$x^2 - x - 4 = 0$$

$$\alpha \quad \beta$$

$$P_n = \alpha^n - \beta^n$$

$$? = \frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$$

$$= \frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$$

$$= \frac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}} = \frac{4P_{13} \cdot 4P_{14}}{P_{13} \cdot P_{14}} = 16$$

$$P_n = \alpha^n - \beta^n$$

$$= \alpha^{n-1} \cdot \alpha - \beta^{n-1} \cdot \beta$$

$$= \alpha^{n-1} (\alpha^2 - 4) - \beta^{n-1}(\beta - 4)\beta^2$$

$$P_n = (\alpha^{n+1} - \beta^{n+1} - 4(\alpha^{n+1} - \beta^{n-1})$$

$$P_n = P_{n+1} - 4P_{n-1} \Rightarrow P_{n+1} - P_n = 4P_{n-1}$$

Q.3 Let $X = \begin{bmatrix}1\\1\\1\end{bmatrix}$ and $A = \begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix}$. For $k \in N$, if $X' A^k X = 33$, then k is equal to ....

3. (10)

$$A = \begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix}$$

$$A^2 = \begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 6\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = I + B, \quad B = \begin{bmatrix}0 & 0 & 6\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

$$A^4 = \begin{bmatrix}1 & 0 & 6\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 6\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \quad B^2 = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix} = 0$$

$$A^{2n} = (I + B)^n$$

$I + nB + 0 + 0 + ...$

$$A^{2n} = I + \begin{bmatrix}0 & 0 & 6n\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix} = \begin{bmatrix}1 & 0 & 6n\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$X'A^k x = [33]$$

$$= [111]A^k\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$= [111]A^{2n}\begin{bmatrix}1\\1\\1\end{bmatrix}(k = 2n)$$

$$= [111]\begin{bmatrix}1 & 0 & 6n\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = [1+0+0 \quad 0+1+0 \quad 6n+0+1]\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$= [1 + 1 + 6n + 1] = [6n + 3] = [33] \; n = 5$$

$$k = 10$$

Q.4 The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is ....

4. (6)

$1012 \le \text{Number in the question} \le 23421$

Also, the number has to use digits {2, 3, 4, 5, 6} without repetition and the number has to be divisible by

$\dfrac{55}{11\times5}$

As the number has to be divisible by both 5 and 11,

$5 \rightarrow$ once place

Let us make 4-digit such numbers first :

{2, 3, 4, 6} (digits are not be repeated)

A number is divisible by 11 it difference of sum of its digits at even places and sum of digits at odd place is 0 or multiple of 11.

$\Rightarrow$ Total 6 numbers 3245, 4235, 6325, 2365, 3465, 6435

Let us make 5 digit such numbers

2 4 3 6 5

but number < 23421

So, no such 5 digit numbers.

Q.5 If $\sum_{k=1}^{10} K^2 \left({}^{10}C_k\right)^2 = 22000$ L, then L is equal to ....

5. **(221)**

$$\sum_{k=1}^{10} k^2 \left({}^{10}C_k\right)^2 = 22000L, L = ?$$

$$r \,.\, {}^nC_r = n \,.\, {}^{n-1}C_{r-1}$$

$$\sum_{k=1}^{10}\left(k \cdot {}^{10}C_k\right)^2 = \sum_{k=1}^{10}\left(10 \cdot {}^{9}C_{k-1}\right)^2 = 100\sum_{k=1}^{10}\left({}^{9}C_{k-1}\right)^2 = 100\sum_{r=0}^{9}\left({}^{9}C_{r}\right)^2$$

$$= 100 \,.\, {}^{18}C_9$$

$$22000L = 100 \,.\, {}^{18}C_9$$

$$L = \frac{{}^{18}C_9}{220} = \frac{(18!)}{11 \times 4 \times 5 (9!)^2}$$

$18! = 2^{16} . 3^8 . 5^3 . 7^2 . 11^1 . 13 . 17$

9 + 4 + 2 + 1	$9! = 2^7 . 3^4 . 5^1 . 7^1$	
6 + 2	4 + 2 + 1	$\frac{18!}{(9!)^2} = 2^2.5.11.13.17$
3+	3 + 1	

= 221

Q.6 If $[t]$ denotes the greatest integer $\leq t$, then the number of points, at which the function $f(x) = 4|2x+3| + 9\left[x+\frac{1}{2}\right] - 12[x+20]$ is not differentiable in the open interval (–20, 20), is ....

6. **(79)**

$f(x) = 4\,|\,2x+3\,| + 9\left[x+\frac{1}{2}\right] - 12[x+20], -20 < x < 20$ doubtful points for differentiability : $x = \frac{-3}{2}$,

$f(x) = 4(2x + 3) + 9(-1) - 12(-2) - 240 = 8x - 213$ for $x = \frac{-3}{2} + h$

$f(x) = -4(2x + 3) + 9(-2) - 12(-2) - 240$

$= 8x - 230$ for $x = \frac{-3}{2} - h$

Not diff. at $x = \frac{-3}{2}$

Other doubtful points : $x+\frac{1}{2}$ = integer

$$-20+\frac{1}{2} < x+\frac{1}{2} < 20+\frac{1}{2}$$

$$-19.5 < x+\frac{1}{2} < 20.5$$

$$x+\frac{1}{2} = -19, -18, ..., 19, 20$$

$x = -19.5, -18.5, -17.5, ..., 18.5, 19.5 \rightarrow$ total 40 numbers.

No. of number = 19.5 – (–19.5) + 1 = 40 (–1.5) included

$-20 < x < 90 \Rightarrow x = -19, -18, ..., 18, 15 \rightarrow$ 39 points

No. of number = 19 – (–19) + 1 = 39

Total : 40 + 39 = 79

Q.7 If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a, b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point (2, –1), then $|2a + 9b|$ is equal to _____ .

7. (195)

$$y = 5x^2 + 2x - 25$$

$P(2, -1)$

$$T_{(p)} : T = 0$$

$\Rightarrow$ $y - 1 = 10x(2) + 2(x + 2) - 50$

$\Rightarrow$ $y = 22x - 45$ is also tangent to $y = x^3 - x^2 + x$ at point (a, b)

For $y = x^3 - x^2 + x$

$$\frac{dy}{dx} = 3x^2 - 2x + 1 > 0$$

$y = 22x - \frac{1939}{27}$ which is not tangent to the curve.

$$3a^2 - 2a + 1 = 22 \text{ (slope of tangent)}$$

$\Rightarrow$ $3a^2 - 2a - 21 = 0 \rightarrow a = \frac{2 \pm \sqrt{4+252}}{6} = \frac{2 \pm 16}{6} = 3, \frac{-7}{3}$

$$b = 27 - 9 + 3 = 21$$

tangent : $y - 21 = 22(x - 3)$

$\Rightarrow$ $y = 22x - 45$

$$a = 3,\ b = 21$$

$$2a + 9b = 6 + 189 = 195$$

Also, $a^3 - a^2 + a = b$

For $a = \frac{-7}{3}$

$$b = \frac{-343}{27} - \frac{49}{9} - \frac{7}{3} = \frac{-343-147-63}{27} = \frac{-553}{27}$$

Q.8 Let AB be a chord of length 12 of the circle $(x-2)^2 + (y+1)^2 = \frac{169}{4}$.

If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to.............. .

8. (72)

Let C be the centre and M be the mid point of AB

$$\triangle APC : \sin\theta = \frac{13/2}{pc} = \frac{5}{13} \Rightarrow pc = \frac{169}{10}$$

$$\triangle AMC : \cos\theta = \frac{6}{13/2} = \frac{12}{13}$$

$$PC = \frac{169}{10}, MC = \frac{13}{2}\sin\theta = \frac{13}{2}\cdot\frac{5}{13}$$

$$PM = PC - MC = \frac{169}{10} - \frac{5}{2} = \frac{144}{10}$$

$$5PM = 72$$

Q.9 Let $\vec{a}$ and $\vec{b}$ be two vectors such that $\left|\vec{a}+\vec{b}\right|^2 = \left|\vec{a}\right|^2 + 2\left|\vec{a}\right|, \vec{a}.\vec{b} = 3$ and $\left|\vec{a}\times\vec{b}\right|^2 = 75$. Then $\left|\vec{a}\right|^2$ is equal to ______.

9. (14)

Given :

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2 \text{ and } \vec{a}\cdot\vec{b} = 3$$

$$|\vec{a}||\vec{b}|\cos\theta = 3$$

$$|\vec{a}|\cos\theta = \frac{3}{\sqrt{6}} = \sqrt{\frac{9}{6}} = \sqrt{\frac{3}{2}}$$

$$|\vec{a}|^2|\vec{b}|^2\sin^2\theta = 75$$

$$|\vec{a}|\sin\theta = \sqrt{\frac{75}{6}} = \frac{5}{\sqrt{2}}$$

$$|\vec{a}|\cos\theta = \sqrt{\frac{3}{2}}$$

$$\Rightarrow \quad \left|\vec{a}\right|^2 = \frac{25}{2} + \frac{3}{2} = \frac{28}{2} = 14$$

Q.10 Let $S = \{(x, y) \in N \times N : 9(x-3)^2 + 16(y-4)^2 \le 144\}$ and $T = \{(x, y) \in R \times R : (x-7)^2 + (y-4)^2 \le 36\}$. Then $n(S \cap T)$ is equal to _____.

10. (27)

$$\frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} \le 1, x, y \in N, (x-7)^2 + (y-4)^2 \ge 36, x, y \in R$$

Total number of common point = 1 + 5 + 7 + 5 + 5 + 3 + 1 = 27

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